Specified Domain in N_u-M_u Interaction Diagram for Logical Judgment in Numerical Analysis on Compression Reinforced Concrete Members

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Abstract: A series of specified domains in N_u-M_u interaction diagram are proposed to determine optimal reinforcement schemes for rectangular RC column sections subjected to uniaxial eccentric load. These domains divide the area covered by interaction diagram into four regions of safety zone, compression-controlled zone, balanced failure zone and tension-controlled zone, which can help engineer to understand all possibilities of section failure under varied reinforcement scheme when they carried out a section design for columns with numerical analysis. With the physical information included in the diagram, the domains help to establish logical judgment between practical reinforcement schemes specified by the Chinese code (GB50010-2010) and corresponding load combination (N_u, M_u) in interaction diagram, and also provide physical interpretation on any calculated result of steel consumption.

Keywords: Failure mode, logical judgment, *N*u-*M*u interaction diagram, specified domains, uniaxial eccentric load.

1. INTRODUCTION

The failure modes of RC members under combined axial load and moment varied from eccentricity of load and arrangement of reinforcement in the tensile side and compressive side through their cross section. Though some numerical analysis methods are employed to set up some charts to meet the need in engineering practice [1], the basic design principle of RC members under uniaxial eccentric load is much more important in helping engineer to build stress mechanism of cross section, control the ultimate limit status of failure and carry out an optimal design finally. Some codes and studies for design of concrete structures provided basic expressions to design RC compression members with rectangular section in asymmetrically reinforcement [2-4]. Since there are three unknowns and only two equilibrium equations, the solution to this kind of problems would be indeterminate if there is no other additional constraint. Some design methods [1, 5] were developed to realize optimum design on column. These methods essentially focused analysis on establishing the ultimate strength of a section with known reinforcement using an iterative design approach, rather than calculating the reinforcement directly to afford specified load combinations, which can be classified as strength checking but strength design. These processes neglect to provide adequately and explicit physical interpretation in section analysis and ignore the influence of reinforcement ratio specified by standards.

In most design of rectangular section columns, the judgment on failure modes of RC columns under eccentric load began from the relative size of eccentricity to depth of column [6]. For a column section, when the eccentricity e_i is small enough, the stress of steel bars farther away from the load is less than their tensile yield strength or even in compression and the strain of compressed fiber of the concrete cross section closed to the load reaches its ultimate value. This failure mode is called compression-controlled failure. When the eccentricity e_i is large enough, steel bars located in tensile side yield in tension before the compressed fiber of the concrete reaches its ultimate compressive strain. This failure mode is so called tension-controlled failure. If tension steel yield at the same time that the concrete on the opposite side reaches its ultimate compression strain, the failure mode is defined as balanced failure (Fig. **1**). In design process, the failure mode even will change with the final reinforcement arrangement across section besides the eccentricity of load.

When a RC column is to be designed or be checked, the first step is to judge possible failure mode by analyzing the eccentricity of load. In some condition, reinforcement scheme is computed by trial and error procedure to ensure which failure mode would be suitable finally, especially for columns with asymmetrically reinforced section. For a symmetrically reinforced column section, though the judgment of failure mode and calculation process is greatly simplified, the comparison between steel bars area of asymmetrical reinforced section and symmetrical reinforced section often brings out meaningful discussion. Moreover, in calculating process, some abnormal results such as negative value in reinforcement lead to confusion and need reasonable interpretation. Some specified domains proposed in N_u-M_u interaction curve can compensate such defects after the initial judgment with the minimum balanced failure eccentricity. The specified domains can also play a guiding role in design or checking of a section, at the same time, make the reinforcement scheme more reasonable and economical.

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2. INTERACTION DIAGRAM FOR COLUMN WITH REASONABLE REINFORCEMENT

The interaction diagram [7] represents the design strength of eccentrically loaded column with known section properties. The curve can demonstrate different destruction path of different slenderness ratio column and give explanation on varied failure mode from material failure of section to unstable failure of whole member. In addition, the curve can help to judge if a given column section is safe to a load effect combination (N, M) and can describe any point on the curve will show compression controlled failure or tension controlled failure (Fig. **1**).

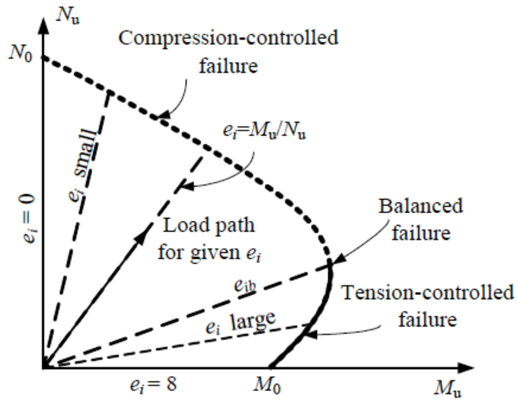

Fig. (1). Different failure modes for column under combined bending and axial load.

For a balanced failure column section (Fig. **2**), The ultimate state equations can be described as follows [3, 4]:

Force equilibrium

$$N_u = \alpha_1 f_c b x_b + f'_y A'_s - f_y A_s \tag{1}$$

Moment equilibrium

$$\begin{aligned} M_u &= N_u e_{ib} \\ &= \alpha_1 f_c b x_b \left(\frac{h}{2} - \frac{x_b}{2}\right) + f'_y A'_s \left(\frac{h}{2} - a'_s\right) + f_y A_s \left(\frac{h}{2} - a_s\right) \end{aligned} \tag{2}$$

$$x_b = \beta_1 x_{nb} = \beta_1 h_0 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} \tag{3}$$

Where f_c is the specified compressive strength of concrete, f'_y is the compressive yielding strength of steel, f_y is the yielding stress of steel, x_b is the depth of equivalent stress block for a balanced rectangular section and α_1, β_1 are two coefficients related to equivalent stress block with section failure.

Introducing $A'_s = \rho' b h_0$, $A_s = \rho b h_0$, and assuming $a_s = a'_s = \chi h_0$, by combining Eqs. (1) and (2) , the eccentricity ratio for balanced failure can be expressed as follows:

$$\frac{e_{ib}}{h_0} = \frac{M_u}{h_0 N_u} = \frac{0.5\left[\frac{x_b}{h_0}\left(1+\chi-\frac{x_b}{h_0}\right)+(1-\chi)\left(\rho'\frac{f'_y}{\alpha_1 f_c}+\rho\frac{f_y}{\alpha_1 f_c}\right)\right]}{\frac{x_b}{h_0}+\rho'\frac{f'_y}{\alpha_1 f_c}-\rho\frac{f_y}{\alpha_1 f_c}} \tag{4}$$

From Eq. (4), when coefficient χ is assumed to be a number as small as possible and the section is reinforced with minimum longitudinal steel ratio in both sides, the minimum eccentricity ratio $e_{ib,min}/h_0$ for a rectangular section at balanced failure can be calculated[6].

Let $\chi = 0.05$,

Then $\rho' = \rho = 1.05 A_s/(bh) = 1.05 \rho'_{min} = 0.21\%$.

For specified material in column section, there is corresponding $e_{ib,min}/h_0$ value, as shown in Table (**1**). Besides the minimum reinforcement ratio, maximum reinforcement ratio of 5% for total cross-section is also specified for compression members to prevent the occurrence of bond damage and concrete cracking under transient unloading [4]. Though maximum steel ratio for one side of cross section is not specified in standard, the ratio of 2.5%, half of 5%, could be deduced reasonably.

From Table **1**, for any rectangular RC column section with normal strength material and cross section, the minimum balanced failure eccentricity ratio $e_{ib,min}/h_0$ is greater than 0.297. It means that for any other section with more reinforcement, if eccentricity of load over depth of section

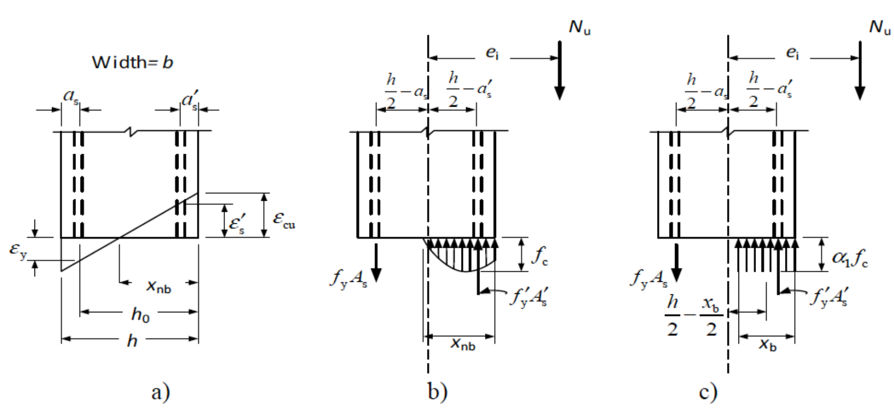

Fig. (2). Balanced failure column subject to eccentric compression. **a**) strain distribution at section, **b**) stresses and forces at balanced failure, **c**)equivalent stress block at balanced failure.

Table 1. Minimum eccentricity ratio $e_{\mathrm{ib,min}}/h_0$ for balanced failure.

Grade of concrete		C30	C40	C50	C60	C70	C80
f_c (MPa)		14.3	19.1	23.1	27.5	31.8	35.9
ε_{cu}		0.0033	0.0033	0.0033	0.0032	0.0031	0.0030
α_1		1.0	1.0	1.0	0.98	0.96	0.94
β_1		0.8	0.8	0.8	0.78	0.76	0.74
HRB335, f_y=300MPa ε_y=0.0015	x_b/h_0	0.550	0.550	0.550	0.531	0.512	0.493
	$e_{\mathrm{ib,min}}/h_0$	0.326	0.307	0.297	0.301	0.307	0.314
HRB400, f_y=360MPa ε_y=0.0018	x_b/h_0	0.518	0.518	0.518	0.499	0.481	0.463
	$e_{\mathrm{ib,min}}/h_0$	0.363	0.339	0.326	0.329	0.334	0.340
HRB500, f_y=435MPa f'_y=410MPa ε_y=0.00218	x_b/h_0	0.482	0.482	0.482	0.464	0.447	0.429
	$e_{\mathrm{ib,min}}/h_0$	0.406	0.375	0.361	0.362	0.365	0.370

e_i/h_0 is less than 0.297, the section must fail in compression controlled mode. The $e_{\mathrm{ib,min}}/h_0 \approx 0.30$ is therefore used as the initial criterion to distinguish between compression failure and tension failure of a column [6].

For a given column cross section under eccentric load, four reinforcement plans in compression and tension sides, $(A'_{s,\min}, A_{s,\min})$, $(A'_{s,\min}, A_{s,\max})$, $(A'_{s,\max}, A_{s,\min})$, $(A'_{s,\max}, A_{s,\max})$, are all reasonable and form the reinforcement scheme boundaries. N_u–M_u interaction diagrams for every boundary reinforcement scheme can be plotted (Fig. **3**). For an axial loaded column, the ultimate states equation can be written as:

$$N_u = \alpha_1 f_c bh + f'_y A'_s + f'_y A_s \tag{5}$$

For reinforcement scheme $(A'_{s,\min}, A_{s,\max})$ and $(A'_{s,\max}, A_{s,\min})$, it can be obtained that $A'_{s,\min} + A_{s,\max} = A'_{s,\max} + A_{s,\min}$. Ignoring influence on section plastic centroid by reinforcement, two interaction diagrams (Fig. **3**) of scheme $(A'_{s,\min}, A_{s,\max})$ and $(A'_{s,\max}, A_{s,\min})$ will have a same significant point *A* which represents the ultimate compression strength with zero moment of the column through Eq. (5).

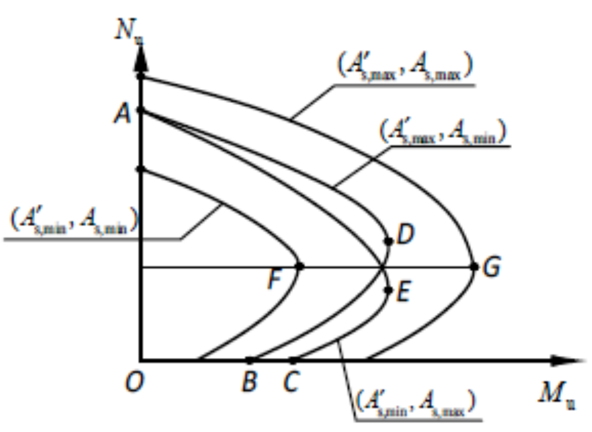

Fig. (3). Interaction diagram of column with maximum or minimum reinforcement ratio.

For a column subject to moment only, the section will fail as a beam. For reinforcement scheme $(A'_{s,\min}, A_{s,\max})$ and $(A'_{s,\max}, A_{s,\min})$, because $A_{s,\max} > A_{s,\min}$, the ultimate flexural capacity of scheme $(A'_{s,\max}, A_{s,\min})$ is less than that of scheme $(A'_{s,\min}, A_{s,\max})$. So point *B* on the curve of scheme $(A'_{s,\max}, A_{s,\min})$ is on the left to point *C* on the curve of scheme $(A'_{s,\min}, A_{s,\max})$.

When column section fail in balanced failure, the depth of compression zone is fixed for an existing section. For column section with reinforcement schemes $(A'_{s,\min}, A_{s,\max})$ and $(A'_{s,\max}, A_{s,\min})$, there is also same depth of equivalent stress block x_b at balanced failure. Assuming $h/2 - a'_s = h/2 - a_s = h_0 - h/2$ (Fig. **2c**), through Eqs. (1) and (2), comparison of balanced failure capacity between two schemes can be expressed as follows:

$$\begin{aligned} &N_{u1} = \alpha_1 f_c b x_b + f'_y A'_{s,\min} - f_y A_{s,\max} \\ &< N_{u2} = \alpha_1 f_c b x_b + f'_y A'_{s,\max} - f_y A_{s,\min} \end{aligned} \tag{6}$$

$$\begin{aligned} &M_{u1} = \alpha_1 f_c b x_b \left(\frac{h}{2} - \frac{x_b}{2}\right) + f'_y A'_{s,\min}\left(\frac{h}{2} - a'_s\right) + f_y A_{s,\max}\left(\frac{h}{2} - a_s\right) \\ &= M_{u2} = \alpha_1 f_c b x_b \left(\frac{h}{2} - \frac{x_b}{2}\right) + f'_y A'_{s,\max}\left(\frac{h}{2} - a'_s\right) + f_y A_{s,\min}\left(\frac{h}{2} - a_s\right) \end{aligned} \tag{7}$$

So balanced point *D* on the curve of scheme $(A'_{s,\max}, A_{s,\min})$ is just above balanced point *E* on the curve of scheme $(A'_{s,\min}, A_{s,\max})$ and the two points are on a vertical line (Fig. **3**).

Points *F* and *G* represent balanced failure points of reinforcement scheme $(A'_{s,\min}, A_{s,\min})$ and $(A'_{s,\max}, A_{s,\max})$ respectively. Through Eq. (1), the two reinforced sections have same ultimate uniaxial load in the failure mode, so point *F* and *G* locate at a horizontal line.

Starting from balanced point *F* of scheme $(A'_{s,\min}, A_{s,\min})$, keeping $A_{s,\min}$ unchanged, increasing compressive steel bars A'_s from $A'_{s,\min}$ gradually, a trajectory of balanced points for varied reinforcement schemes can be plotted in interaction diagram. Through Eqs. (1) and (2), increment of N_u and M_u can be obtained respectively:

$$\begin{aligned}\Delta N_u &= [\alpha_1 f_c b x_b + f_y'(A'_{s,\min} + \Delta A'_s) - f_y A_{s,\min}] \\ &\quad -[\alpha_1 f_c b x_b + f_y' A'_{s,\min} - f_y A_{s,\min}] \\ &= f_y' \Delta A'_s\end{aligned} \tag{8}$$

$$\begin{aligned}\Delta M_u &= [\alpha_1 f_c b x_b (\frac{h}{2} - \frac{x_b}{2}) + f_y'(A'_{s,\min} + \Delta A'_s)(\frac{h}{2} - a'_s) \\ &\quad + f_y A_{s,\min}(\frac{h}{2} - a_s)] \\ &\quad -[\alpha_1 f_c b x_b (\frac{h}{2} - \frac{x_b}{2}) + f_y' A'_{s,\min}(\frac{h}{2} - a'_s) \\ &\quad + f_y A_{s,\min}(\frac{h}{2} - a_s)] \\ &= f_y' \Delta A'_s (\frac{h}{2} - a'_s)\end{aligned} \tag{9}$$

Then

$$\Delta M_u / \Delta N_u = \frac{h}{2} - a'_s \tag{10}$$

Eq.(10) shows that trajectory of balanced points of every reinforcement scheme from $(A'_{s,\min}, A_{s,\min})$ to $(A'_s, A_{s,\min})$ is a straight line with a constant slope of $h/2 - a'_s$ and the trajectory travel to the balanced point D of scheme $(A'_{s,\max}, A_{s,\min})$ finally. In a similar way, another three trajectories from scheme $(A'_{s,\min}, A_{s,\min})$ to $(A'_{s,\min}, A_{s,\max})$, $(A'_{s,\max}, A_{s,\min})$ to $(A'_{s,\max}, A_{s,\max})$ and $(A'_{s,\min}, A_{s,\max})$ to $(A'_{s,\max}, A_{s,\max})$ can also be figured out and corresponding physical interpretation is shown in Fig. (**4**). For example, line *FE* is trajectory of balanced points from scheme $(A'_{s,\min}, A_{s,\min})$ to $(A'_{s,\min}, A_{s,\max})$ and its slope is $-(h/2 - a_s)$. Among the four trajectories, line *FD* is parallel to *EG* and so *FE* to *DG*. The four lines form a specified domain *FDGE*. Fig. (**4**) possesses the following physical information:

1) The interaction diagram with specified domains can help to judge the properties of rectangular section column with fixed dimension and make a most reasonable choice in much optional reinforcement schemes in section design.
2) The column section subject to load combinations (N_u, M_u) covered by specified domain in diagram can de designed as balanced failure mode with reasonable reinforcement that codes allowed.
3) The slope of line from origin of coordinate *O* to any point (N_u, M_u) covered by the specified domain is corresponding balanced failure eccentricity, $e_{ib} = M_u / N_u$, and slope of line *OFQ* is minimum balanced failure eccentricity for most material strength arrangement. Line *OFQ* locates on the upper left to specified domain through graphics analysis. This can also be testified by the value of $\Delta M_u / \Delta N_u = h/2 - a'_s$ (Eq. (10)) which is generally greater than the value of $e_{ib,\min} / h_0$ (Table **1**).
4) Balanced failure points for reinforcement scheme with same value of $A'_s + A_s$ will locate on a vertical line in specified domain, while balanced failure points for reinforcement scheme with same value of $A'_s - A_s$ form a horizontal line in the domain.
5) In specified domain, any line parallel to boundary line *FD* represents trajectory of balance failure points for column section with same A_s which satisfy inequality $A_{s,\max} \geq A_s \geq A_{s,\min}$ and any line parallel to boundary line *FE* represents trajectory of balance failure points for column section with same A'_s.
6) The curve *KFB* is interaction diagram of section with minimum reinforcement ratio. So any load combinations (*N*,*M*) falls inside the region surrounded by curve *KFB* and two coordinate axes is reliable for any section with any reinforcement ratio permitted by standard.
7) Load combinations (*N*,*M*) in surrounded region through points *FKHQGDF* can't find reasonable reinforcement arrangement to reach balanced failure mode or tensile-controlled failure and result in a compression-controlled failure eventually.
8) Load combinations (*N*,*M*) in surrounded region through points *FBCGEF* will result in tension controlled failure with section reinforcement allowed by standard.
9) If calculated reinforcement area in any side of section is negative, at least one side reinforcement should be arranged with minimum ratio whenever the section will destruct in tension failure or compression failure.

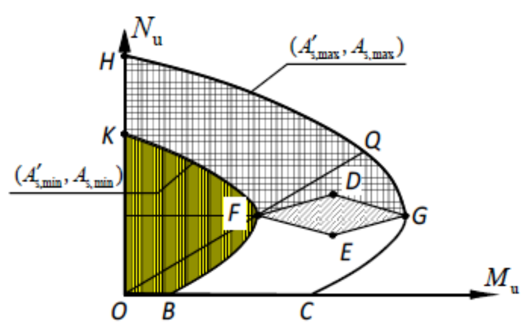

Fig. (4). Specified domains in interaction diagram.

3. THE DESIGN OBJECTIVE AND OPTIMAL DESIGN USING SPECIFIED DOMAIN

In design practices, most reinforced concrete columns are symmetrically reinforced for convenience in construction or to meet capacity demand under cyclic loads. However, for some cases, such as the retaining walls or frame columns in low intensity seismic area or columns of portal rigid frames in which the moments are uniaxial and the eccentricity is large, it is more economical to use an asymmetrical pattern of bars.

For a given section and the load combination (*N*,*M*), there are much kinds of interaction curves getting through the point (*N*,*M*) in a section's diagram with specified materials. So there are various reinforcing schemes to meet strength demand of (*N*,*M*). Under the consideration of ductile failure requirements and minimum material consumption, if the eccentricity allows, reinforcement should be arranged to realize tension controlled failure or at least balanced failure as far as possible. When the values of (N,M) can only lead to a compression controlled failure, it should also be considered firstly to adopt an effective reinforcement scheme which means the minimum steel consumption.

As mentioned above, keeping value of $A'_s + A_s$ as a constant, the balanced failure points for these reinforcement schemes locate on a vertical line in specified domain (Fig. **5**). When (N,M) located in the domain, the scheme of minimum reinforcement area $A'_s + A_s$ can be obtained by assuming the depth of equivalent stress block $x_b = \beta_1 x_{nb}$ and the section will be designed as balanced failure. From three cases shown in Fig. (**5**), curve of case 1 represents interaction diagram of reinforcing scheme which compression steel area A'_s is greater than tension steel area A_s. Curve of case 2 is for steel arrangement of $A'_s = A_s$ and curve of case 3 is for steel arrangement of $A'_s < A_s$. Because curve of case 1 is above to the other two curves in compression zone (Fig. **5**), the scheme of $A'_s > A_s$ has more effective strength for compression controlled failure and reinforcing arrangement of $A_s = A_{s,\min}$ under the condition of constant value of $A'_s + A_s$ can play the most effective role. Correspondingly, the arrangement of $A'_s < A_s$ has more effective strength for tension controlled failure and reinforcing scheme of $A'_s = A'_{s,\min}$ under the condition of constant value of $A'_s + A_s$ can play the most effective role. When A'_s for compression failure or A_s for tension failure respectively reaches the maximum reinforcement ratio specified by standard, the more effective reinforcing scheme is to increase A_s in $(A'_{s,\max}, A_s)$ for compression controlled failure or increase A'_s in $(A'_s, A_{s,\max})$ for tension controlled failure respectively. Therefore, the four boundary lines of the specified domain *FDGE* are the most excellent choice of reinforcement scheme for unbalanced failure.

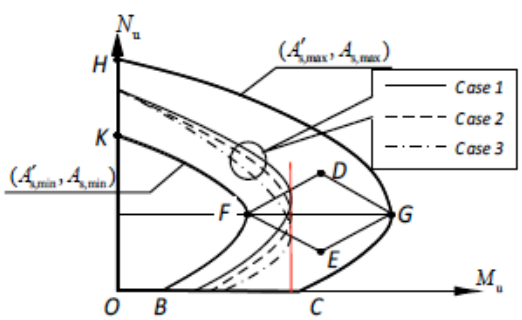

Fig. (5). Optimal design employing specified domain in interaction diagram.

4. LOGICAL JUDGEMENT AND DESIGN PROCEDURE AIDED BY SPECIFIED DOMAIN IN INTERACTION DIAGRAM

In normal design process, the eccentricity of load e_i is calculated firstly and compared to minimum balanced failure eccentricity $e_{ib,\min}$. If $e_i < e_{ib,\min} = 0.30h_0$, the section must be designed under compression controlled failure. If $e_i > 0.30h_0$, the section is assumed to be a balanced failure member in advance and the depth of equivalent stress block $x_b = \beta_1 x_{nb}$. For the moment equilibrium, Eq. (2) can be written as:

$$N_u(e_i + 0.5h - a_s) = \alpha_1 f_c b x_b (h_0 - \frac{x_b}{2}) + f'_y A'_s (h_0 - a'_s) \quad (11)$$

Putting in $e_i = M_u / N_u$, Eq. (11) can be written as:

$$M_u + N_u(0.5h - a_s) = \alpha_1 f_c b x_b (h_0 - \frac{x_b}{2}) + f'_y A'_s (h_0 - a'_s) \quad (12)$$

Then

$$A'_s = \frac{M_u + N_u(0.5h - a_s) - \alpha_1 f_c b x_b (h_0 - \frac{x_b}{2})}{f'_y (h_0 - a'_s)} \quad (13)$$

$$A_s = \frac{\alpha_1 f_c b x_b + f'_y A'_s - N_u}{f_y} \quad (14)$$

If above calculated result, $A_s, A'_s \in (A_{s,\min}, A_{s,\max})$ with $e_i > 0.30h_0$, the corresponding (N_u, M_u) will locate in specified domain and value of A_s, A'_s are eventually adopted. If not, logical judgment was introduced to develop following design process:

1) If $A'_s \in (A_{s,\min}, A_{s,\max})$, but $A_s < A_{s,\min}$, the (N_u, M_u) will fall in compression failure zone *FKHQGDF* (Fig. **4**) or safety zone. $A_s = A_{s,\min}$ should be taken as initial value in tension side to calculate compressive steel area A'_s.
2) If $A'_s > A'_{s,\min}$, the section will be designed as a compression failure mode.
3) If $A'_s < A'_{s,\min}$ or even if A'_s is negative, let $A'_s = A'_{s,\min}$.
4) If $A'_s \in (A_{s,\min}, A_{s,\max})$, but $A_s > A_{s,\max}$, the (N_u, M_u) will fall in tension failure zone(Fig. **4**). $A_s = A_{s,\max}$ should be regarded as initial value to work out steel area A'_s.
5) If $A'_s \le A'_{s,\max}$, then adopt the calculation results.
6) If $A'_s > A'_{s,\max}$, then increase the section dimension or improve material strength.
7) If $A'_s < A'_{s,\min}$ or even if A'_s is negative, A'_s should initialized to $A'_{s,\min}$ to work out A_s.
8) If $A_s \le A_{s,\min}$, then reinforce the section by minimum reinforcement ratio;
9) If $A_{s,\min} < A_s \le A_{s,\max}$, then adopt the calculation results;
10) If $A_s > A_{s,\max}$, then let $A_s = A_{s,\max}$ to compute A'_s in turn; if the new value $A'_s > A'_{s,\max}$, increase the section dimension or improve material strength.
11) If $A'_s > A'_{s,\max}$, the (N_u, M_u) will fall in compression failure zone(Fig. **4**). Let $A'_s = A'_{s,\max}$ to work out A_s.
12) If $A_s \le A_{s,\min}$, let $A_s = A_{s,\min}$ to work out A'_s in compression failure mode;
13) If $A_{s,\min} < A_s \le A_{s,\max}$, then adopt the calculation results;
14) If $A_s > A_{s,\max}$, then reselect section or improve the strength of material.

All the logical judgments above are with certainty, and according to the calculation results, there is no need to checking relevant depth of compression stress x/h_0 on the basis of reinforcement that calculated.

CONCLUSION

For a given section of eccentrically loaded member, any reinforcement scheme that conforms to standards and regulations has a corresponding N_u-M_u interaction diagram. The specified domain proposed in this paper can plan out the corresponding destruction zones for different (N, M) in more detail. After initial judgment with minimum balanced failure eccentricity ratio, specified domain can be employed to serve for guide the design process between various failure modes. The final failure mode can be certainly judged and the physical meanings can be visually and concisely obtained, which make it easier for engineers or programmer to understand and grasp the design process.

To understand the exist of specified domain in N_u-M_u interaction diagram, reinforcement influence on plastic centroid of section is ignored provisory. In addition, there is no need to pay attention to actual shape and position of the specified domain. Just according to logic concept contained by the specified domain, engineers can clearly find the practical process for asymmetric reinforcement design of rectangular cross section. Visual comparison between asymmetric and symmetrical reinforcement schemes can also be taken in an interaction diagram. At the same time, the concept of specified domain also has better reference to physical interpretation of strength checking on an existing section's reinforcement.

CONFLICT OF INTEREST

Financial contributions to the work being reported should be clearly acknowledged, as should any potential conflict of interest.

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