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# About the Reliability of Punching Verifications in Reinforced Concrete Flat Slabs

F. Porco[1,*], G. Uva[1], M. Sangirardi[1] and S. Casolo[2]

[1]*Dipartimento DICATECh, Politecnico di Bari, Italy;* [2]*Dipartimento ABC, Politecnico di Milano, Italy*

**Abstract:** Reinforced concrete slabs are a widely diffused structural solution either in Italy, or abroad; this for a series of advantages connected to their structural conception and their performances. However, this series of advantages is obtained as a result of proper design, especially oriented to appropriately sizing the thickness of the plate itself. Moreover, for flat structures with concentrated loads, as the case of flat slabs on punctual supports, phenomenon of punching can't be neglected, as it inevitably affects the structure and so it must be taken into account even in the early stages of the project. In this paper an attempt to evaluate the reliability of punching verifications has been made, referring to some in force regulations; this has been possible making a comparison between the mean resisting value of punching, obtained applying law prescription and the real collapse load for some columns belonging to a building that collapsed during the early stages of its construction. For the specific case study analyzed, there has been the opportunity to perform an on-site investigation, collecting a great amount of information regarding the mechanical properties of the used materials, the real positioning of the rebars in the structural elements, the real amount of concrete cover and so on. Since in a punching failure mechanism a crucial element is the resistance of the concrete, the precise definition of its properties attains great importance, especially when existing buildings are involved. To analyze the exact conditions of collapse and the factors that may have originated it, an important procedure is the acquisition of information, as much as possible, about the mechanical properties of the onsite materials. After this first evaluation of the accuracy of the actual regulations, another interesting comparison has been carried out, studying the influence of two main factors in the definition of the punching shear resistance; more specifically, it has been observed which is the contribution of the variability of the compressive strength of the concrete on site and the effective depth of the plate. The first related to a series of circumstances that may affect the value of the resistance, reducing it from that established during the design phase, the second closely related to inaccuracy in the laying of the longitudinal reinforcement. Therefore two sensitivity analysis have been performed, either referring to Eurocode 2, or following the prescriptions given in Model Code 2010, varying once the mean value of the compressive strength of the concrete, once the effective depth d.

**Keywords:** Reinforced concrete slabs, design codes, punching shear resistance, reliability analysis.

## 1. INTRODUCTION

Flat slab structures are, nowadays, widespread as a common structural solution for residential and office buildings. The main reasons that account for this are the following: they are an economical structural system, they simplify and speed up construction operations and they guarantee a flexible space partitioning.

Though all these advantages, the flat slab systems present a complex behavior, especially for what concerns the joint area, that is in the slab–column connection.

Some of the main problems which affect flat slabs concern with deformations and cracking, mostly due to stresses which derive from self weights and dead loads. Regardless of which design method is used, the resulting slab must be serviceable at the working load level, with deflections and cracking remaining within acceptable limits. Slab design methods are concerned largely with flexure, but shear forces may also be a limiting factor.

*Address correspondence to this author at the Dipartimento DICATECh, Politecnico di Bari, via Orabona 4, 70126 Bari, Italy; Tel: +39 080 5963832; Fax: +39 080 5963832; Email: f.porco@poliba.it

Reinforced concrete slab floors have taken many forms since their introduction, but slabs may generally be divided into two broad categories: beamless slabs and slabs supported on beams located on all sides of each panel; of course there are many hybrid variants. Beamless slabs are described by the generic terms *flat planes* and *flat slabs*. The flat plate is an extremely simple structure in concept and construction, consisting of a slab of uniform thickness supported directly on columns. The flat plate is a direct development from the earlier flat slab structure which was characterized by the presence of capitals at the tops of the columns and usually also by drop panels or thickened areas of the slab surrounding the column.

The choice between the use of flat slabs and flat plates is largely a matter of the magnitude of the design loading and of the spans. The strength of the flat plate structure is often limited by the strength in punching shear at sections around the columns, and they should be consequently used with light loads [1].

Reinforced concrete flat plate slabs and slabs with drop panels often exhibit radial cracking in the vicinity of column supports under normal service/construction loading. This behavior has been observed in slabs in which design and/or construction errors have been identified, and in properly designed and constructed slabs. So, the occurrence of radial cracking is not itself indicative of either design of construction errors, much less unanticipated performance.

While this cracking may result in undesirable exposure of the negative flexural reinforcing to moisture and chlorides, it is normally not indicative of a serious structural problem, and can be readily shown by analysis to occur under normal service conditions [2].

One of the most common phenomena that are involved in is punching. The punching failure mechanism results from the simultaneous presence of shear and flexural stresses at the edge of the column and it is associated with the formation of a pyramidal plug of concrete which punches through the slab.

In some cases, punching strength is insufficient due to several reasons, such as change of the building use, design and/or construction errors, corrosion of reinforcement and deterioration of concrete, leading to the necessity of repair and/or strengthen the structure.

Punching shear failure, a brittle failure mode, is the major disadvantage of this structural system. It occurs with almost no warning signs because deflections are small and cracks at the top side of the slab are usually not visible. A local punching failure at one column will result in increased shear force at surrounding columns which can trigger the punching failure to the adjacent columns resulting in the progressive collapse of the complete structure.

Punching is a local failure mechanism and, since it mainly involves the concrete, it is also a brittle failure mechanism. This premature failure mode can happen in structures such as bridge deck slabs, often subjected to severe conditions of loads concentrated in small areas, and characterized by a reduced thickness, in flat slabs used as foundations subjected to the concentrated loads transferred by columns, or in flat r.c. slabs used as horizontal elements in the areas of load introduction at column-slab connection.

Despite the fact that punching failure takes place in a circumscribed area it can be the origin of a progressive collapse, and in some cases a global structural collapse. In fact, when a support in a slab–column connection is lost, there is an increase of stresses in the nearby slab–column connections and their probability of failure is enhanced [3].

The term progressive collapse has been used to describe the spreading of an initial local failure within a structure, which can lead to partial or total collapse of the structure in a manner analogous to a chain reaction. The local failure is triggered by the loss of one load carrying member. Following the initial failure, the structure seeks alternative load paths to transfer the load originally carried by the damaged portions to the adjacent undamaged members. As the latter may or may not have adequate strength to withstand the additional loads, further redistribution of loads are likely to occur until an equilibrium state is reached. However, due to the magnitude of the loads involved, equilibrium may only be achieved when a substantial part of the structure has already collapsed. Therefore, the main feature of progressive collapse is that the final damage is disproportionately larger than the local damage that initiated the collapse.

Taking the cue from a real case of collapse of a reinforced concrete slab, in which a punching shear mechanism occurred, the purpose of this study is to compare the different formulations suggested by national and international regulations and to perform a sensitivity analysis referring to some relevant parameters such as concrete strength, reinforcement, concrete cover, which may affect the final value of punching resistance.

The objective of this study is to evaluate the reliability of punching verifications. This has been made on the basis of a real case study on which a on-site survey has been carried out, in order to determine the "real" mechanical properties of the constituent materials, and an analytical calculus of the punching resistance according to law prescriptions, following both Eurocode and Model Code. The obtained results have then been compared with the "collapse value" really exhibited by the structure.

So two themes have been combined: the first is connected to the definition of the real mechanical properties of materials cast on site, since sometimes they are considerably different from those stated in the previous design calculations, both for poor quality of construction, and for an influence of environmental conditions which inevitably affect the resistance features of the structure as a whole.

The design of the experimental on site survey, with a precise idea of the distribution of the test and of the position of the samples, may have a determining role in the definition of the performance of an entire structural existing system.

The second point, which has already been widely discussed in the previous lines, is the problem of "punching resistance" of concrete structures sensitive to this phenomenon; in this context, there are two main difficulties that make this issue particularly interesting: the difficulty of studying a phenomenon whose definition and modeling is still an open question in the scientific community and the complexity of the definition of the on-site mechanical properties of materials by destructive and non-destructive techniques.

## 2. PUNCHING: STATE OF THE ART AND RECENT DEVELOPMENTS

Reinforced concrete slabs on columns were initially developed in the U.S. and Europe at the beginning of the 20th century. Their designs typically included large mushroom-shaped column capitals to facilitate the local introduction of concentrated loads from the slab to the column. In the1950s, flat slabs without capitals started to become widespread. Because of their simplicity, both for their construction and for their use (simple formwork and reinforcement, flat soffit allowing an easy placement of equipment, and installation underneath the slab), they have become very common for medium height residential and office buildings as well as for parking garages.

The design of flat slabs is mostly governed by serviceability conditions on the one side (with relatively large deflections in service) and by the ultimate limit state of punching shear (also called two-way shear) on the other side. These two criteria typically lead to the selection of the appropriate slab thickness.

The area of the slab in the immediate vicinity of the column, especially edge and corner columns, is subjected to a complex triaxial stress state influenced by many mutually dependent parameters. Although a significant number of tests have been carried out, mostly on reinforced, cast in place, concrete slab-internal column connections, yet there is no general analytical solution for the punching problem.

Most of the mechanical models used to study the phenomenon are based either on the assumption that the load is transferred by inclined compression struts (one of the first and the most famous is “Kinunen and Nilander model”) or on purely empirical “critical section” approach. Recently, an increasing number of mechanical models based on a different shear transfer action have been proposed. In these models, the concrete tensile strength and post-cracking behavior of concrete play an important role. For example, in the model proposed by Menétrey [4], the punching strength is governed by concrete tie strength (strut and tie analogy for load transfer), obtained by summing the vertical component of residual tensile stresses around the punching crack.

Punching shear has been the object of an intense experimental effort since the 1950s and in most cases, the phenomenon is investigated by considering an isolated slab element.

This element typically represents the surface of the slab surrounding a column and is delimited by the line of contraflexure for radial moments, which are zero at a distance $r_s \approx 0.22L$ (according to a linear-elastic estimate), where $L$ is the axis-to-axis spacing of the columns. In recent years, several state-of-the-art reports and synthesis papers have been published on this topic [5-7]. Most design codes base their verifications on a critical section, with the punching shear strength of slabs without shear reinforcement defined as a function of the concrete compressive strength and often of the reinforcement ratio.

In the early 1960s, Kinnunen and Nylander [8] tested a series of slabs in punching, varying amongst other parameters the amount of flexural reinforcement in the slab. The following observations can be made from the load-rotation relationships of the tests:

- For low reinforcement ratios (test with ρ = 0.5%), the observed behavior is ductile, with yielding of the entire flexural reinforcement, as illustrated by the horizontal asymptote of the load-rotation curve. In this case, the strength of the slab is limited by its flexural capacity and punching occurs only after large plastic deformations. The punching failure at the end of the plastic plateau remains brittle and leads to a sudden drop in strength;
- For intermediate reinforcement ratios (tests with ρ = 1.0%/0.5% and 1.0%), some yielding of the reinforcement is present in the immediate vicinity of the column, but punching occurs before yielding of the entire slab reinforcement. In this case, the strength of the slab is lower than its flexural capacity;
- For large reinforcement ratios (test with ρ = 2.1%/1.0%), punching occurs before any yielding of the reinforcement takes place, in a very brittle manner. In this case, the strength of the slab is significantly lower than its flexural capacity;
- Increasing the reinforcement ratio increases the punching capacity, but strongly decreases the deformation capacity of the slab.

On the basis of their test results, Kinnunen and Nylander [8] developed a rational theory for the estimate of the punching shear strength based on the assumption that the punching strength is reached for a given critical rotation ψ. This rotation was calculated by simplifying the kinematics of the slab and assuming a bilinear moment-curvature relationship. This proposal remains one of the best models for the phenomenon of punching. Recently, some improvements were proposed by Hallgren [9] and Broms [10] to account for size effects and high strength concrete.

While very elegant and leading to good results, this model was never directly included in codes of practice because its application is too complex. It served as a basis, however, for the Swedish and Swiss design codes of the 1960s.

In 2008 Muttoni proposed a new failure criterion for punching shear [11] based on the critical shear crack theory. This criterion describes the relationship between the punching shear strength of a slab and its rotation at failure, it is consistent with the works of Kinnunen and Nylander and it accounts for size effect.

The critical shear crack theory states that the punching shear strength decreases when the rotation of the slab increases; this has been explained by Muttoni and Schwartz [12] as follows: the shear strength is reduced by the presence of a critical shear crack that propagates through the slab into the inclined compression strut carrying the shear force to the column.

Some evidences supporting the role of the shear critical crack in the punching shear strength are the following:

1. It has been shown experimentally [8, 13] that the radial compressive strain in the soffit of the slab near the column, after reaching a maximum for a certain load level, begins to decrease. Shortly before punching, tensile strains may be observed. This phenomenon can be explained by the development of an elbow-shaped strut with a horizontal tensile member along the soffit due to the development of the critical shear crack. A similar phenomenon has been observed in beams without shear reinforcement [14];

2. Experimental results by Bollinger [15] also confirm the role of the critical shear crack in the punching strength of slabs. The tested slab was reinforced by concentric rings placed at the boundary of the slab element only. With this particular reinforcement layout, only radial cracks developed and the formation of circular cracks in the critical region was avoided. Thus, the punching shear strength of this test was significantly larger than that of a similar slab with an additional ring in the critical region. For this test, the presence of an additional ring in the vicinity of the critical region initiated the development of a crack in that region, with a subsequent reduction of the punching shear strength of approximately 43%.

Design rules for punching shear present in design codes are generally based on experimental results performed on isolated slab elements representing the part of the slab close to the column.

Most tests have been performed on relatively thin slabs, typically 0.1 to 0.2 m. The test results are commonly extrapolated to design flat slabs with a thickness typically 2 to 3 times larger, and even for foundation mats with thicknesses 10 to 20 times larger.

A paper published in 2008 [11] proposes a mechanical model based on the critical shear crack theory, explaining punching behavior of flat slabs without shear reinforcement and correctly accounting for size effect.

According to the proposed failure criterion, the punching strength is a function of the opening of a critical shear crack in the slab. This failure criterion simultaneously determines the punching load and the rotation capacity of the slab, and thus its ductility.

Even if tests on thin slabs have exhibited some level of ductility for low reinforcement ratios, the behavior is quite brittle for thicker slabs and the only solution to reach a satisfactory level of ductility is to place punching shear reinforcement.

## 3. TECHNICAL RULES

Most of the codes regarding punching shear verifications, prefer not to consider the complex theories that analyze the phenomenon, but they bring back the punching verification to a verification of the shear strength made for a conventional surface.

According to Italian Standards NTC 2008 [16] plates should be verified against punching, at ultimate limit state, when subjected to concentrated loads. The evaluation should be done at the points of load application and at the columns' sections.

When no reinforcement against shear stresses had been provided, the resistance of the entire slab is empowered to the tensile strength of concrete.

Conversely when punching reinforcement is provided, the entire strength is absorbed by it at the ultimate limit state.

However, this code doesn't give an analytic formulation, but recommends to refer to standards of proven validity.

The European design code Eurocode 2 [17] (EC2) proposes empirical design equations for estimating the punching shear strength in slabs without transverse reinforcement; they are the same equations applied for one-way shear. The punching shear strength per unit length is assumed to be constant for the entire control perimeter around internal columns with balanced moments. The EC2 formulas are written in terms of the concrete compressive strength, the reinforcement ratio in both orthogonal directions ( $\rho_x$ and $\rho_y$ ) and the size effect factors.

Punching shear can result from a concentrated load or reaction acting on a relatively small area, called the *loaded area* $A_{load}$ of a slab or a foundation.

An appropriate model is shown in (Fig. **1**).

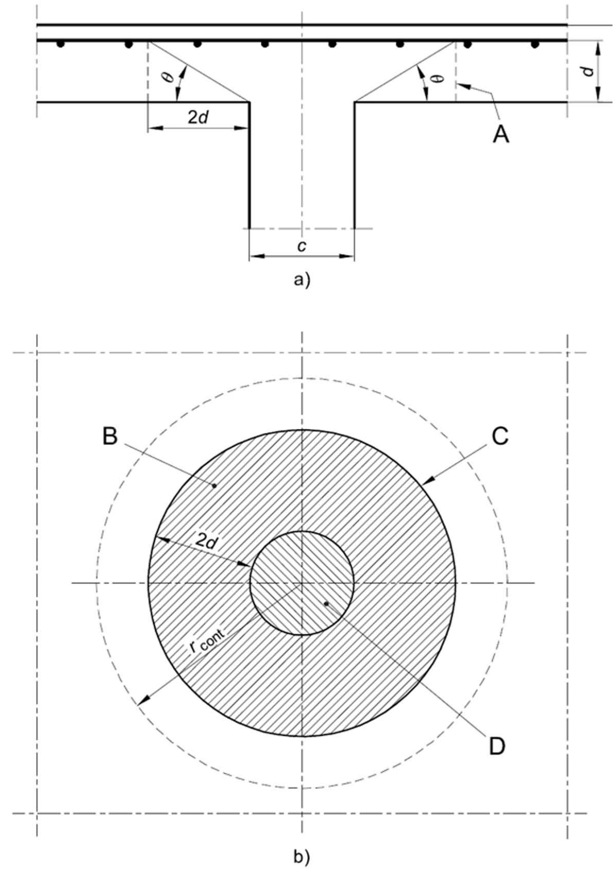


**Fig. (1).** Model for Eurocode 2 prescriptions.

Where:

$\theta = \arctan\left(\frac{1}{2}\right) = 26{,}6°$

$A$ – Basic control section

$B$ – Basic control area $A_{cont}$

$C$ – Basic control area perimeter $u_1$

$D$ – loaded area $A_{load}$

$r_{cont}$ – further control perimeter

The shear resistance should be checked at the face of the column, at its perimeter $u_0$ and at the basic control perimeter $u_1$. If shear reinforcement is required a further perimeter $u_{out,ef}$ should be found where shear reinforcement is no longer required. The rules given by Eurocode 2 are principally formulated for the case of uniformly distributed loading. The basic control perimeter $u_1$ may normally be taken to be at a distance $2.0\ d$ from the loaded area and should be constructed so as to minimize its length; the effective depth of the slab is assumed constant and may normally be taken as:

$$d_{eff} = \frac{(d_x + d_y)}{2}$$

where $d_x$ and $d_y$ are the effective depths of the slab in two orthogonal directions.

For a loaded area near an edge or a corner, the control perimeter should be taken as shown in (Fig. **2**), if this gives a perimeter (excluding the unsupported edges) smaller than that obtained from the expressions above.

The control section is that which follows the control perimeter and extends over the effective depth *d*. For slabs of constant depth, the control section is perpendicular to the middle plane of the slab. For slabs or footings of variable depth other than step footings, the effective depth may be assumed to be depth at the perimeter of the loaded area.

The two perimeters, in correspondence of which the checking should be made, are calculated as following:

$$u_0 = 2(b_x + b_y)$$

$$u_1 = 2(b_x + b_y) + 2\pi(2d)$$

Knowing the active punching force $V_{Ed}$ we can calculate the effective punching stress at the edge of the column:

$$v_{Ed} = \frac{V_{Ed}\beta}{u_0 d}$$

where $\beta$ is a factor which depends on the eccentricity of the loading, or rather on the position of the column in the plane of the slab, as shown in (Fig. **3**) for example:

$$\begin{cases} \beta = 1 & internal\ column \\ \beta = 1.4 & edge\ column \\ \beta = 1.5 & corner\ column \end{cases}$$

At this stage the first check is fulfilled if results:

$$v_{Ed} \leq v_{Rd,max} = 0.5\nu f_{cd} \quad [\nu = 0.5]$$

Then we can calculate the punching shear stress at a distance of $2d$, with the following expression:

$$v_{Ed} = \frac{\beta V_{Ed}}{u_1 d}$$

And then it's possible to make a further check:

$$v_{Ed} \leq v_{Rd,c}$$

The design punching shear resistance [MPa] may be calculated as follows:

$$v_{Rd,c} = C_{Rd,c} k (100\rho_l f_{ck})^{1/3} \quad \geq v_{min}$$

where

- $f_{ck}$ is in MPa
- $k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$
- $\rho_l = \sqrt{\rho_{lx}\rho_{ly}} \leq 0.02$

$\rho_{lx}$ and $\rho_{ly}$ are related to the flexural reinforcement in *x* and *y* direction respectively. The values $\rho_{lx}$ and $\rho_{ly}$ should be calculated as mean values taking into account a slab width equal to the column width plus *3d* each side.

N.B. : The values of $C_{Rd,c}$ , $v_{min}$ for use in a country may be found in its National Annex. The recommended value (in Italy) for $C_{Rd,c}$ is $\frac{0.18}{\gamma_c}$ and for $v_{min}$ is $0.035k^{(3/2)} f_{ck}^{1/2}$.

In the Fib - Model Code 2010 [18] the design shear force for what concerns punching is calculated as the sum of design shear forces acting on a basic control perimeter $b_1$. This basic control perimeter may normally be taken to be at a distance $0.5d_v$ from the support region or loaded area and should be constructed so as to minimize its length. The length of the control perimeter is limited by slab edges. The effective depth of the slab ($d_v$) shall account for the effective level of the support region.

For the calculation of the punching shear resistance, a shear-resisting control perimeter ($b_0$) is used. For a general case, perimeter $b_0$ can be obtained on the basis of shear fields as:

$$b_0 = \frac{V_{Ed}}{v_{perp,d,max}}$$

where $v_{perp,d,max}$ is the maximum value of the projection of the shear force perpendicular to the basic control perimeter.

Approximate rules may be applied for calculation of the shear-resisting control perimeter.

The punching shear resistance must be calculated as:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s}$$

where the design shear resistance attributed to the concrete may be taken as:

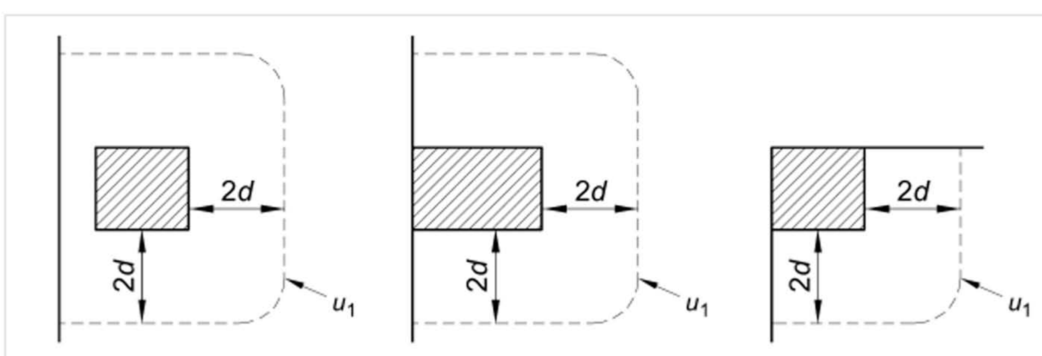


**Fig. (2).** Definition of control perimeter for internal, edge or corner columns.

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v$$

with $f_{ck}$ in [MPa].

The parameter $k_\psi$ depends on the deformations of the slab around the support region and is calculated as:

$$k_\psi = \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dg}} \leq 0.6$$

where $d$ is the mean value [mm] of the effective depth in the $x$ and $y$ directions, and the parameter $\psi$ refers to the rotation of the slab around the support region outside the critical shear crack.

The calculation of the rotation $\psi$ is different according to which level of approximation has been chosen. In the discussed case study, the verification has been made following the second level of approximation, since the first is more appropriate during the preliminary design and gives a too conservative estimate of the resistance of the slab. For more information see [18].

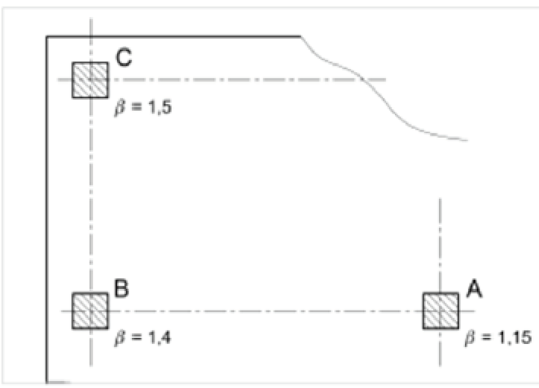


**Fig. (3).** Eccentricity factors, Eurocode 2.

## 4. THE CASE STUDY

### 4.1. General Description

The study carried out in this paper focuses on the case of a collapse, during its construction, of a reinforced concrete building, entirely cast on site, with columns as vertical bearing elements, plates as horizontal elements, realized with in-situ reinforced concrete as well and continuous beam foundations.

The thickness of the horizontal plates varies at the different altitudes; it is 30 cm at a height of +5.9m and +9.45 m, 25 cm at the coverage floor and 20 cm at the height of 12.45 m.

The collapse involved one of the two blocks in which the entire building was divided, while there was taking place the casting of the concrete for the realization of the second floor.

After the breakdown of the underlying slab, realized approximately one month before, the entire block collapsed; the remaining portion of the building was not affected by the event.

The flooring type adopted was the common concrete plate entirely cast on site, with an expected in plane extension of 585 $m^2$ and a thickness of 30 cm, resulting a total volume of about 150 $m^3$.

The dynamic of the collapse can be approximately reconstructed as follows: the first floor plate first yielded, then, detached from the columns, slipped almost along them, while the reinforcing bars shear off or slipped out the column-slab connection.

As can be noticed in (Fig. **4**), there are some similarities between the theories by Muttoni and the evidences observed, concerning the shape of the punching "cone" and the failure mechanism.

Analyzing a series of information collected, it is possible to assess the following elements:

- Collapse occurred while the second floor plate was still being realized, in particular when 2/3 of the total area had been cast.
- Collapse had been announced by a series of warning signs, such as: noises, rubble falling, bearing elements' cracking, noticeable deformation of the horizontal structures, yielding of the reinforcement.
- At the initial deformation of the slab, followed its "slippage" along the columns. The floor, almost undamaged, lied on the ground, having swept away during its fall all the underlying scaffolds, while the columns generally remained standing. The entire event happened in a relatively gradual way.

It is nearly sure that the "local" cracking in correspondence of the column-slab connections, was followed by the collapse of some vertical elements strongly bended, that were literally pulled along by the plate.

Once the central bearing elements failed, those which sustained the slab, the other columns all around were loaded by an unexpected portion of the plate, with a cantilever mechanism, and subsequently by a heavy bending stress, for which they were obviously not designed.

In fact, right in this area, columns appear partially or totally overturned, while some portions of the slab still result completely attached to the columns.

### 4.2. Experimental Investigations

As it has been said before, in the analysis of the existing structures, a crucial element is represented by the recognition of the mechanical properties of the materials.

For what concerns the structure analyzed in the present study, the mechanical properties stated in the technical report are the following:

***Concrete***

C25/30 for the foundations

C28/35 for structures in elevation

***Steel***

FeB44K

Since the first results were collected, it appeared clearer and clearer that, concerning the investigation of the mechanical properties of the materials, the crucial factor was

**Fig. (4).** Some slab-column connection details after the collapse.

the concrete. So a general investigation planning has been drawn up in order to define, as accurately as possible, the properties of the concrete cast on site.

#### *4.2.1. Investigation Plan*

An investigation plan, regarding destructive and non-destructive tests to perform either in the collapse area, or on that part of the structure that still stood in place after the event, choosing which element to investigate, the type and an indicative number of the tests to be performed [19-21] has been drawn up.

The purpose of this plan was essentially to investigate the concrete mechanical properties on site; it is in fact well known that problems related to the degree of compaction, aging and in general to the conditions of casting and laying may alter the mechanical behavior and, particularly, the mechanical strength of a concrete structure can be even significantly different from that measured by laboratory tests on samples prepared and aged according to law prescriptions.

The strategy adopted was to take a precise number of samples by core drilling from structural elements, to be related with a more extensive database of results obtained by a series of non destructive tests (concrete hammer test and ultrasonic test).

In the following (Table **1**) some relevant data have been summarized; in the first column an indication of the location of the sample has been given. In fact B2 indicates that the core belongs to the so called Block 2, one of the two structures which composed the entire building studied. In the last columns, Vs indicates the velocity measured in the ultrasonic test, Vc indicates the same velocity, but adjusted in order to consider site-effects. The parameter I indicates the result of the concrete hammer test which is the well known rebound index, while fcc and Rcc indicate the compressive strength derived from the tests, cylindrical and cubic respectively.

### 4.3. Processing of the Experimental Results

The results of non destructive tests have been related to the mechanical resistance of the samples taken from the existing structures, that have been subjected to crushing tests. In particular three correlations have been carried out:

1. Correlation between mechanical strength and ultrasonic speed
2. Correlation between mechanical strength and concrete hammer rebound index
3. Correlation between mechanical strength, ultrasonic speed and concrete hammer rebound index (SonReb method)

These correlations have been obtained by a statistical processing of the experimental data available on the samples, applying the least square method to identify the best interpolation curve.

The relations that have been used are the following:

1. $R = A \cdot B^V$
2. $R = A \cdot B^I$
3. $R = A \cdot B^V \cdot C^I$

Where:

$R$ = Concrete compressive strength

$A, B, C$ are constants to be determined on the basis of experimental values

$V$ = ultrasonic speed

$I$ = rebound index

At this point it is possible to evaluate the in situ-strength value, on the basis of non destructive measures, by the previous relations; values obtained have been considered as

**Table 1. Main Features of the Cores**

| Core Drill Id | Height | Diameter | Vs | Vc | I | fcc | Rcc |
|---|---|---|---|---|---|---|---|
| | mm | mm | m/s | m/s | | MPa | MPa |
| B2 P42 BD/1 | 188.6 | 94.3 | 4154 | 3880 | 41 | 23.01 | 28.76 |
| B2 P42 BD/2 | 187.3 | 94.3 | 4175 | 3810 | 40 | 20.92 | 26.1 |
| B2 P35 BD/1 | 185.3 | 94.3 | 4036 | 3600 | 40 | 14.69 | 18.28 |
| B2 P35 BD/2 | 187.2 | 94.3 | 4020 | 3580 | 40 | 14.86 | 18.54 |
| B2 P31 BD/1 | 188.1 | 94.3 | 3810 | 3580 | 40 | 16.37 | 20.45 |
| B2 P31 BD/2 | 187.5 | 94.3 | 3788 | 3610 | 40 | 15.21 | 19.98 |
| B2 P37 BD/1 | 187.2 | 94.3 | 4000 | 3680 | 40 | 22.85 | 28.51 |
| B2 P37 BD/2 | 187.4 | 94.3 | 4016 | 3680 | 40 | 17.01 | 21.23 |
| B2 P23 AC/1 | 189.0 | 94.3 | 4170 | 3890 | 49 | 24.00 | 30.02 |
| B2 P23 AC/2 | 188.6 | 94.3 | 4195 | 3890 | 49 | 21.21 | 26.51 |
| B2 P27 BD/1 | 188.6 | 94.3 | 4324 | 3900 | 46 | 19.10 | 23.88 |
| B2 P27 BD/2 | 186.3 | 94.3 | 4381 | 3890 | 45 | 31.79 | 39.62 |
| B2 P47 BD/1 | 187.5 | 94.3 | 3017 | 3060 | 32 | 14.80 | 18.47 |
| B2 P47 BD/2 | 186.5 | 94.3 | 3040 | 3070 | 32 | 9.41 | 11.73 |

"sampling resistance". The abundance of data (non destructive analysis have been performed in proper quantity) allows a statistic manipulation, obtaining a "characteristic value of the on-site resistance".

Values obtained, however, can't be directly compared with characteristic design value, for a series of reasons that make the on-site resistance physiologically lower than the "potential design" one, ensured only by a sample manufactured and kept in particular conditions of humidity and temperature, in a word, in ideal conditions.

From many experiences reported in literature, an adequate and sufficiently conservative choice is to take a characteristic design value as follows:

$$R_{ck(effective)} = 1.25 \cdot R_{ck(situ)}$$

This methodology has been applied on the data collected, obtaining the following regression curves:

1. $R = A \cdot B^V \; A = 1.00054614 \;\; ; \;\; B = 2.6465139$
2. $R = A \cdot B^I \quad A = 1.04276797 \;\; ; \;\; B = 4.0800615$
3. $R = A \cdot B^V \cdot C^I$
   $A = 1.00041 \;\; ; \;\; B = 1.013206 \;\; ; \;\; C = 2.639652$

The application of these curves at the experimental results' data-base has provided "sample resistances" and the statistical manipulation of those values allowed to obtain final values of the "characteristic compressive strength in-situ" and, subsequently, the design strength too.

Values of the effective compressive strength obtained with the three different methods are very similar: this fact, despite the small number of data used for the statistical manipulation, for the great homogeneity observed, enables to consider them sufficiently reliable (Table **2**).

We can finally state that the concrete on site has a compressive strength whose value is the average between the three values above, that's to say: $f_{cm,site} = 22.576\ MPa$.

To confirm the value obtained, since the experimental results of laboratory tests were available, another correlation has been performed, between mass density of a well compacted specimen taken during the cast and matured in laboratory ( $m_{v0}$) and the mass density of the core taken directly from the structure.

It has to be remarked that there is a difference between the mechanical compression resistance of a specimen reproduced in laboratory and that of a specimen taken from the structure, mainly because the compaction degree is different in one case and in the other.

Calling $g_c$ the compaction degree, defined as:

$$g_c = \frac{m_v}{m_{v0}}$$

An indicative estimate of the percentage resistance is provided by this relation for the most common cases ($0.9 \le g_c \le 1$):

$$\Delta R = 500 \cdot (1 - g_c)$$

That is, for each centesimal point of compaction degree with respect to unit, a 5% less in mechanical resistance is registered, compared with the value obtained for the laboratory specimen.

**Table 2. General Overview**

| General overview | | | |
|---|---|---|---|
| **Correlation** | **N° of Samples** | **Population** | **Effective Rck [Mpa]** |
| Ultrasonic Analysis | 14 | 37 | 23.05 |
| Concrete Test Hammer | 14 | 76 | 22.06 |
| SonReb Method | 14 | 37 | 22.62 |

In (Table **3a**) the data belonging to the samples prepared picking up the concrete during the casting operation, well compacted and aged in laboratory are summarized, while (Table **3b**) contains the data (for what concerns geometry, mass and density) of the core drills taken on site, so they're the same mentioned in (Table **1**), on which the non destructive tests have been performed.

Using the data collected, the average compaction degree is:

$$g_c = 0.93$$

Consequently, the resistance variation is:

$$\Delta R = 33.3\%$$

This value, that indicates a decrease of the cylindrical strength from $28\ ^{N}/_{mm^2}$ to $22.6\ ^{N}/_{mm^2}$, is substantially aligned with in situ investigations. This circumstance can be explained by an inaccurate setup of the construction.

### 4.4. Numerical Simulations and Identification of the Collapse Conditions of the Structure

In order to evaluate the stresses in the stage of incipient collapse, the calculation has been performed considering the first floor slab loaded only by the weight of the second floor slab.

The evaluation of the resulting actions on the considered columns has been made using a finite element method, using conventional *beam* and *shell* elements. The most relevant difficulties arose in the identification of the effective loads in which the collapse occurred.

The applied load condition includes dead loads and the permanent overload due to that part of the second floor slab that had been cast at the time of the collapse.

During the casting operations, the upper storey doesn't represent a resisting element, but just an overloading, having an effect on the first floor.

Other accidental loadings, of course, have not been included, since the load conditions were completely known when the collapse occurred and, there are no safety factor acting on that loadings.

In this situation, the security checks relevant according to the phenomenon occurred, are the punching ones, on which we're going to focus, since they are important to understand the dynamics of the structural collapse.

These checks have been performed using the stresses determined on the basis of the calculus of the structural model described above, using various methods, according to more recent regulations, Italian and international ones.

The verification procedures adopted are those following the Italian NTC 2008 [16], which are substantially aligned with the European Code, Eurocode 2 and the fib Model Code 2010.

In the following paragraphs references have been made only to Eurocode and Model Code; as said in section 3, Italian regulations don't provide any analytical formulation to verify slabs against punching and so verifications are usually made using, as a reference, Eurocode 2.

The most important verifications are those made on the slab at 5.9 m height and the relative sustaining columns (see Fig. **5**).

#### *4.4.1. Numerical Results*

Two verifications have been therefore performed, following the two above-mentioned set of rules, referring to Eurocode 2 and Model Code 2010, adopting the second level of approximation for the determination of the rotation $\psi$, whose results are more precise and less conservative.

All the data were available for few columns and so the precise estimate of the collapse loading has been performed just for these elements; in this context it has to be remarked that the situation is analogous to that of an existing structure, and so all the data concerning mechanical properties of materials have to be intended as average values. In the following (Table **4a**, **4b**, **4c**) and (Fig. **6**), the most relevant results are summarized.

Aiming at clarifying the values adopted for the main parameters involved in the computation some relevant input data are summarized for the three columns A, B and C.

Since there are no specific reinforcement to absorb tensile stresses, verifications have been carried out considering only the concrete as resisting element, with the contribution of the flexural reinforcement, as allowed in the followed law prescriptions.

Having on-site resistance lower than those declared in the technical report, underlines the great uncertainty of the entire life cycle of concrete structures. These uncertainties are usually secured by the material safety factor; so, for this reason, in the previous verifications, this factor has been eliminated; furthermore, it has to be remarked that all the uncertainties connected to the processes of production, transportation and laying can't be removed not even through on-site surveys, and so many uncertainties still remain about the fortuitous conditions at the moment of collapse.

It has to be remarked that the collapse loading, which had caused the failure of the entire structure, has been derived from an *a posteriori* estimate; however, though taking account of this whole series of uncertainties, the obtained results can be considered acceptable.

So, it is possible to state that the previsions included in the actual regulations, represent an excellent estimate of the punching collapse loading.

**Table 3a. Geometric Properties of the Samples**

| Sample Id | Dimension 1 | Dimension 2 | Dimension 3 | Mass | Density |
|---|---|---|---|---|---|
| | mm | mm | mm | g | kg/mc |
| 1F | 152 | 150 | 150 | 7700 | 2251 |
| 1F | 152 | 150 | 150 | 7640 | 2234 |
| 2F | 153 | 150 | 150 | 7800 | 2266 |
| 2F | 152 | 150 | 150 | 7760 | 2269 |
| 3F | 151 | 150 | 150 | 7740 | 2278 |
| 3F | 151 | 150 | 150 | 7760 | 2284 |
| 1SP | 152 | 150 | 150 | 7700 | 2251 |
| 1SP | 152 | 150 | 150 | 7720 | 2257 |
| 2SP | 150 | 150 | 151 | 7730 | 2275 |
| 2SP | 150 | 150 | 152 | 7730 | 2260 |
| 2SP | 150 | 152 | 150 | 7730 | 2260 |
| 2SP | 150 | 152 | 150 | 7770 | 2272 |
| 3SP | 154 | 150 | 150 | 7780 | 2245 |
| 3SP | 154 | 150 | 150 | 7990 | 2306 |
| 4SP | 153 | 150 | 150 | 7870 | 2286 |
| 4SP | 153 | 150 | 150 | 7930 | 2304 |
| Average value | | | | | 2269 |

**Table 3b. Geometric Properties of the Cores**

| Core Drill Id | Height | Diameter | Mass | Density |
|---|---|---|---|---|
| | mm | mm | g | kg/mc |
| B2 P42 BD/1 | 188.6 | 94.3 | 2833.6 | 2151.2 |
| B2 P42 BD/2 | 187.3 | 94.3 | 2774.5 | 2121.0 |
| B2 P35 BD/1 | 185.3 | 94.3 | 2771.6 | 2141.6 |
| B2 P35 BD/2 | 187.2 | 94.3 | 2756.1 | 2108.0 |
| B2 P31 BD/1 | 188.1 | 94.3 | 2687.7 | 2045.9 |
| B2 P31 BD/2 | 187.5 | 94.3 | 2747.6 | 2098.2 |
| B2 P37 BD/1 | 187.2 | 94.3 | 2815.8 | 2153.7 |
| B2 P37 BD/2 | 187.4 | 94.3 | 2788.4 | 2130.5 |
| B2 P23 AC/1 | 189.0 | 94.3 | 2779.3 | 2105.5 |
| B2 P23 AC/2 | 188.6 | 94.3 | 2832.0 | 2150.0 |
| B2 P27 BD/1 | 188.6 | 94.3 | 2894.1 | 2197.1 |
| B2 P27 BD/2 | 186.3 | 94.3 | 2822.8 | 2169.5 |
| B2 P47 BD/1 | 187.5 | 94.3 | 2664.4 | 2034.6 |
| B2 P47 BD/2 | 186.5 | 94.3 | 2646.2 | 2031.6 |
| Average Value | | | | 2117.0 |

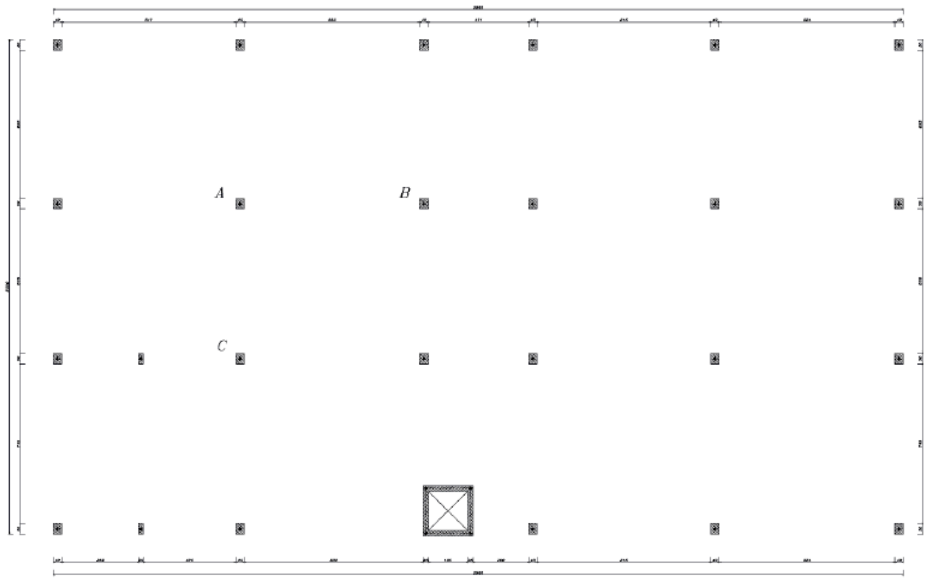


**Fig. (5).** General overview of the plan.

**Table 4a. Eurocode2, Summary of Relevant Values and Results**

| Col. | Eurocode 2 | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Vsd | dx | dy | d | Lx | Ly | u0 | ix | iy | Asx | Asy | ρx | ρy | ρ | k | vrd | Vrd | **Vsd/Vrd** |
| | kN | mm | mm | mm | mm | mm | mm | mm | mm | mmq | mmq | - | - | - | - | Mpa | kN | **%** |
| A | 820.42 | 270 | 250 | 260 | 400 | 500 | 5065.6 | 1960 | 2060 | 5306.6 | 4239 | 0.0104 | 0.0079 | 0.0091 | 1.88 | 0.925 | 1218.2 | **77.4** |
| B | 612.91 | 270 | 250 | 260 | 400 | 500 | 5065.6 | 1960 | 2060 | 4270 | 2995.56 | 0.0084 | 0.0056 | 0.0068 | 1.88 | 0.842 | 1108.8 | **63.6** |
| C | 909.1 | 270 | 250 | 260 | 400 | 500 | 5065.6 | 1960 | 2060 | 3925 | 4239 | 0.0077 | 0.0079 | 0.0078 | 1.88 | 0.880 | 1158.5 | **90.2** |

**Table 4b. Model Code 2010, SUMMARY of Relevant Values and Results**

| Col. | Model Code 2010 | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Vsd | d | Lx | Ly | rsx | rsy | bs | msd | mrd | ψx | Kψ | kdg | b0 | fcm | Vrd | **Vsd/Vrd** |
| | kN | mm | mm | mm | mm | mm | mm | kN | kN | - | - | - | mm | Mpa | kN | **%** |
| A | 820.42 | 260 | 8620 | 7260 | 1896.4 | 1597.2 | 2611 | 102.5525 | 200.2681 | 0.009021 | 0.324 | 0.75 | 2354.8 | 22.6 | 944.0 | **86.9** |
| B | 612.91 | 260 | 8620 | 7260 | 1896.4 | 1597.2 | 2611 | 76.61375 | 163.424 | 0.007902 | 0.346 | 0.75 | 2354.8 | 22.6 | 1008.2 | **60.8** |
| C | 909.1 | 260 | 8620 | 7260 | 1896.4 | 1597.2 | 2611 | 113.6375 | 204.0597 | 0.01023 | 0.303 | 0.75 | 2354.8 | 22.6 | 883.2 | **102.9** |

**Table 4c. Summary of Results**

| | | $f_{cm}$ = 22.6 MPa (on site) | |
|---|---|---|---|
| **Column ID** | **Acting Force** | **Action/Resistance Ratio [%]** | |
| | **[kN]** | **EC2** | **MC10** |
| A | 820.4 | 77.4 | 86.9 |
| B | 612.9 | 63.6 | 60.8 |
| C | 909.1 | 90.2 | 102.9 |

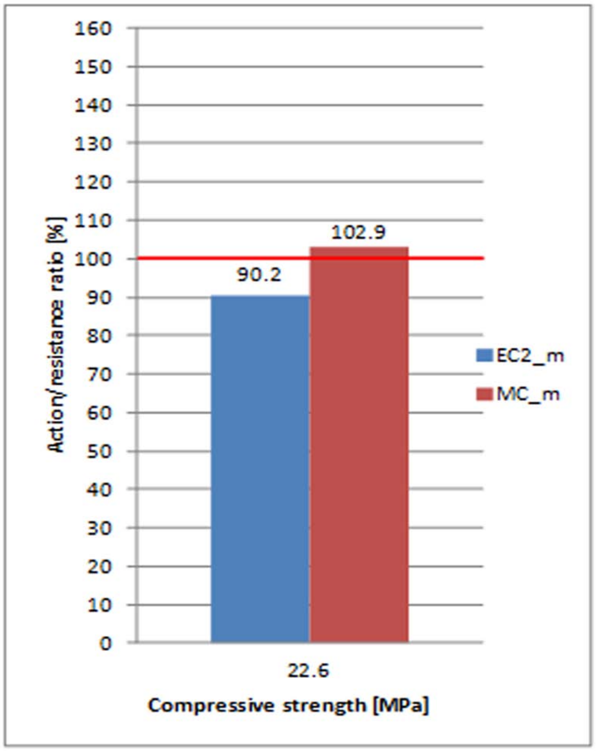


**Fig. (6).** Action-Resistance ratio according to Eurocode 2 and Model Code 2010. Left: Design resistance, Right: mean values detected on site.

In the light of what has been said in the previous paragraphs, it's not difficult to understand that, once the punching collapse load is reached for that column, failing at that point the "support" of that portion of the slab, the other surrounding columns are found to be encumbered by a load they were not designed for, and therefore the structure has been affected by a mechanism of chain collapse.

## 5. SENSITIVITY ANALYSIS

Additional considerations may result from some sensitivity analysis on the basis of the two formulations considered in the previous calculations.

The formulas above have been applied again, keeping all the parameters unchanged, except for one, whose influence has been therefore evaluated.

The two parameters studied are: the average concrete compressive strength and the effective depth of the slab *d*. The objective is to evaluate once the influence of the quality of the packaging and of the laying of the concrete, once the accuracy in the positioning of the longitudinal reinforcement.

As already mentioned in the previous paragraphs, the concrete is characterized by a resistance susceptible to considerable variation, linked to multiple factors, including the mix design, the packaging and transportation processes, laying operations, the environmental conditions of maturation; for what concerns the variability of the effective depth, it has been evaluated according to the possible variation of the design concrete cover, thus assuming a certain tolerance we proceeded to calculate the punching resistance for each value of d in the range between 220 and 290 mm, having as a useful height in situ a value equal to 260 mm (average value of the useful height in the two directions *x* and *y*).

In the figures above, some "significant" values are represented. In (Fig. **7**), analyzing the influence of the effective depth *d*, it has been highlighted the value of an effective depth obtained with a tolerance of *10 mm*, aligned with the in force law prescriptions (Eurocode 2).

In (Fig. **8**), analyzing the influence of the average compressive strength, the marked values are respectively:

- the average value of the concrete compressive strength detected on site, intended as the minimum value allowed, in agreement with what is specified in current Italian regulations [16].
- the average value of the compressive strength, calculated according to the following expression:

  $f_{cm\ situ} = 0.85\ f_{cm,design}$

  since the average value of the resistance on site is generally lower than that measured on samples collected during the casting operations and matured in ideal conditions.

Italian regulations allow to obtain an average value of compressive strength, measured with appropriate techniques (destructive and non-destructive) not less than 85% of the average value defined in the design phase.

What appears clear observing the figures above, is that both factors involved in this analysis play a determining role; their variation can cause a consistent reduction of the punching shear resistance, both following Eurocode 2, and Model Code 2010.

An interesting consideration arises from the combined examination of the (Figs. **7** and **8**): one can easily observe that, while remaining within the limits allowed by current regulations, changes in the compressive strength of the concrete and in the effective depth of the slab, as a result of physiological errors in the realization of a reinforced concrete structure, can cause reductions in punching resistance of approximately 12% in the case of the calculation carried out by following Eurocode 2, and about 19% in the case of the calculation carried out in agreement with Model Code 2010.

## 6. CONCLUSIONS

This paper is aimed at giving an overview on the in force law prescriptions for what concerns punching. In all codes punching shear capacity calculations are based on a critical perimeter, which is located between *0.5* and *2d* from the face of the column. In both codes examined the punching shear capacity depends on the flexural reinforcement ratio, though it's influence is quite different in each code. Within this framework the study presented has focused on two main topics: the first connected with the determination of the effective mechanical properties of materials, specifically of the concrete, in an existing reinforced concrete building; the second topic connected to the reliability of the punching verifications suggested by the actual regulations. In this sense, analyzing Eurocode and Model Code formulations applied to a real case study, it has been possible to prove that they give a good estimate of the collapse loading.

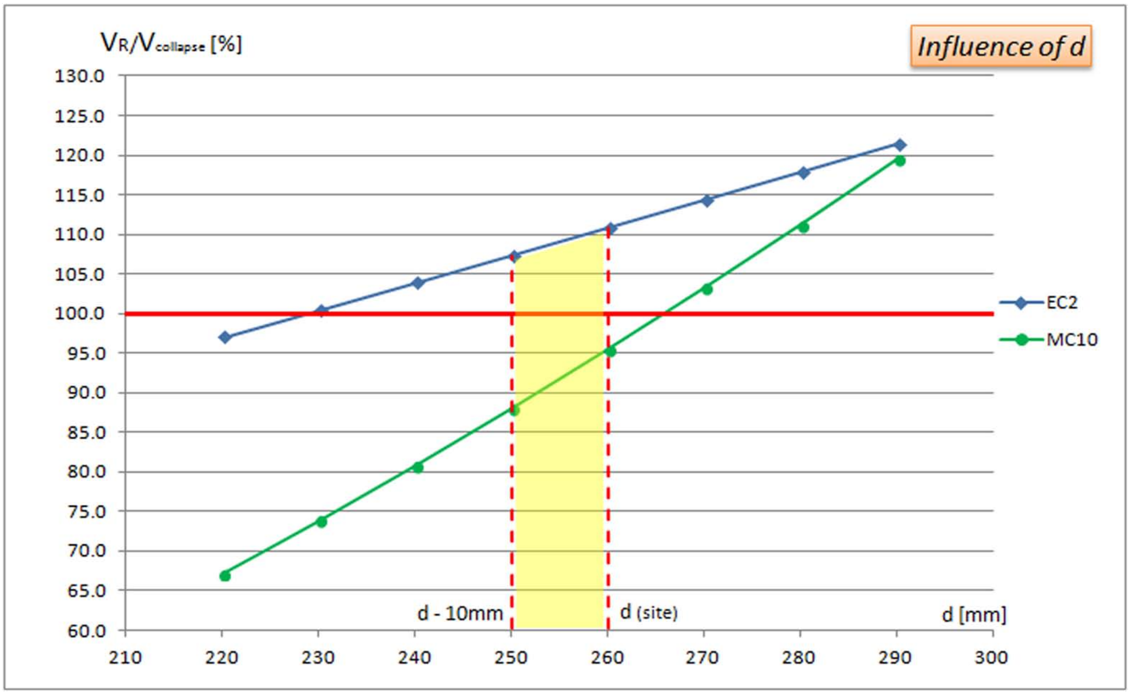


**Fig. (7).** Influence of d according to Eurocode 2 and Model Code 2010.

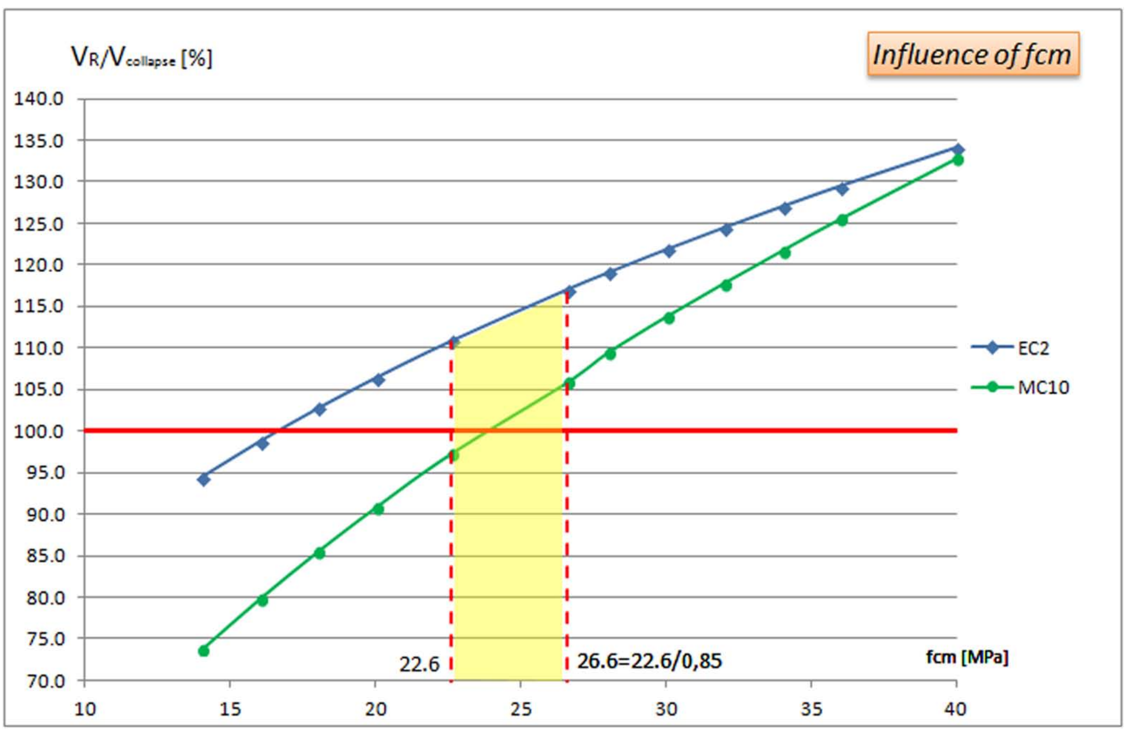


**Fig. (8).** Influence of fcm according to Eurocode 2 and Model Code 2010.

The chance to analyze a great amount of data directly derived from a real case of a collapse in a structure, so with no scale effects, has underlined a good correspondence between observed values and previsions given by current regulations.

However, in the last part of the manuscript, the attempt was to underline how some parameters, whose variability is sometimes out of designers' control in the realization stages, can heavily influence punching resistance.

In this context, the sensitivity analysis carried out have demonstrated the importance of the "quality" both in the design of the mixture, and in the realization of the structure itself; in fact, though remaining within the tolerances allowed by the mentioned regulations, either for what concerns average compression strength of the conglomerate, or for the effective depth, there could be a significant drop in the resulting value of the punching resistance.

## CONFLICT OF INTEREST

The author(s) confirm that this article content has no conflicts of interest.

## ACKNOWLEDGEMENT

The research presented in this article was partially funded by the Department of Civil Protection, Project ReLUIS-DPC 2010-2013.

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Received: June 12, 2013 Revised: July 27, 2013 Accepted: July 31, 2013