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Dynamic Identification Techniques to Numerically Detect the Structural Damage

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Abstract: Damage detection in civil engineering structures using changes in measured modal parameters is an area of research that has received notable attention in literature in recent years. In this paper two different experimental techniques for predicting damage location and severity have been considered: the Change in Mode Shapes Method and the Mode Shapes Curvature Method. The techniques have been applied to a simply supported finite element bridge model in which damage is simulated by reducing opportunely the flexural stiffness *EI*. The results show that a change in modal curvature is a significant damage indicator, while indexes like MAC and COMAC – extensively and correctly used for finite element model updating - lose their usefulness in order to damage detection.

Keywords: Structural damage detection, dynamic identification, mode shapes, modal curvatures.

1. INTRODUCTION

Most road and railway bridges and viaducts are, at least in Europe, 30-40 years old and have been designed when the problems of structural durability had not yet been taken into account. This circumstance, together with an insufficient maintenance and an incorrect functioning – due to the continuous increase of traffic - is at the origin of their current decay (more or less evident).

In literature with the term *damage* is indicated any change introduced into a system that negatively affects its current or future performances [1]. The process of implementing a damage detection strategy for civil engineering structures is called *Structural Health Monitoring.* Several research works have pointed out the importance to develop techniques able either to locate damage or to determine its influence on the global mechanical behavior of a structure. Most currently used damage detection methods are visual or localized methods using acoustic, ultrasonic, magnetic field, X-ray or thermal principles. All of these experimental techniques require that the location of the damage is a-priori known and that the portion of the structure to be inspected is easily accessible, therefore allowing to detect a possible damage only near the surface of the structure. Recently several techniques of *dynamic identification* have been developed. These techniques are able to get and examine changes in the vibration characteristics of a structure. The basic premise is that damage alters the stiffness, mass or energy dissipation properties of a structure; they in turn alter the measured structural dynamic response. As a consequence, the experimental response of a damaged structure during a dynamic investigation moves away from the expected one.

Structural health monitoring can be described as a process composed of four levels, which altogether also represent the state of the art of damage detection by vibration monitoring:

1. Periodically execution of dynamic investigations and comparisons of the dynamic characteristics extracted from measurements (eigenfrequencies, mode shapes and modal damping) with the analogous ones relative to the initial undamaged state, in order to determine the presence of damage in the structure [2].
2. Definition of modal parameters sensible to damage, that is parameters obtained from the extracted dynamic characteristics and able to precisely locate the damage position. This is the most significant field of research, in order to define damage identification methods that are at the same time reliable and easily feasible. In this paper the indexes MAC [3] and COMAC [4] and the modal curvatures [5] are considered.
3. Quantification of damage severity by updating a finite element (FE) model [6]. In such a procedure the uncertain properties (e.g. stiffness distribution) in the FE model are adjusted by minimizing the differences between the measured modal parameters (eigenfrequencies and mode shapes) and those numerically obtained from FE predictions.
4. Evaluation of the remaining structural integrity and risk assessment; this point is still characterized by a certain approximation, because it regards either the risk connected with the structural strength decrease or the entire life-cycle cost of the infrastructure, involving an approach at the same time technical and economical to the problem of infrastructures maintenance [7, 8].

In reference to point 1 that actually described the steps adopted in an Operation Modal Analysis (OMA) different methods and techniques have been proposed that operate in the time and frequency domains. Often the input is not

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clearly known, so that *output-only procedures* are utilized, such as Stochastic Subspace Identification (SSI) in the time domain and Enhanced Frequency Domain Decomposition (EFDD) in the frequency domain. These methods have also been implemented in softwares such as ARTEMIS [9] and LEONIDA [10].

The aim of this paper is the comparison of two damage detection methods proposed in literature: the Change in Mode Shapes Method, based on MAC and COMAC indexes, and the Mode Shapes Curvature Method, based on modal curvatures. A similar procedure that considers the evaluation of MAC and COMAC indexes in the way described in this paper, is utilized in the OMA of ancient structures assumed as prototypes of the way of building in the historical centers of the South of Italy and the Ionian Islands of Greece [11, 12] (Interreg SMART BUILT project founded under the European Cooperation Programme Greece-Italy 2007-2013 "Investing in our future").

2. DAMAGE IDENTIFICATION METHODS

2.1. Change in Mode Shapes Method

The Change in Mode Shapes Method is based on the comparison between two series of mode shapes deducible from the results of two different dynamic investigations performed on a same structure. The comparison can be carried on calculating MAC and COMAC indexes, afterwards described.

The following relation [5] determines the Modal Assurance Criterion (MAC):

$$MAC_{(j,k)} = \frac{\left(\sum_{i=1}^{n} \Phi_{Aj}^{i} \times \Phi_{Bk}^{i}\right)^2}{\sum_{i=1}^{n}\left(\Phi_{Aj}^{i}\right)^2 \times \sum_{i=1}^{n}\left(\Phi_{Bk}^{i}\right)^2} \quad \text{with:} \quad \begin{array}{l} j=1,\ldots,m_A \\ k=1,\ldots,m_B \end{array} \tag{1}$$

where: Φ_A and Φ_B are two series of mode shapes expressed in matrix form (eigenmode matrices), respectively of ($n\times m_A$)- and ($n\times m_B$)-class, with m_A and m_B equal to the number of investigated modes and n equal to the number of considered coordinates (that is the number of measurement points); Φ^{i}_{Aj} is the *i*th coordinate of the *j*th column of Φ_A (that is the *j*th vibrational mode), while Φ^{i}_{Bk} is the *i*th coordinate of the *k*th column of Φ_B (that is the *k*th mode of vibration).

Those symbols can be justified remembering the form assumed by the eigenmode matrices Φ_A and Φ_B:

$$[\Phi_A] = [\{\Phi_{A1}\}\ \{\Phi_{A2}\}\ \ldots\ \{\Phi_{Aj}\}\ \ldots\{\Phi_{Am}\}]$$

$$[\Phi_B] = [\{\Phi_{B1}\}\ \{\Phi_{B2}\}\ \ldots\ \{\Phi_{Bj}\}\ \ldots\{\Phi_{Bm}\}]$$

where $\{\Phi_{Aj}\}$ and $\{\Phi_{Bj}\}$ are vectors of n components (in number equal to the measurement points). It has been supposed that the two data sets under consideration ($[\Phi_A]$ and $[\Phi_B]$) have the same number m of investigated mode shapes.

For a fixed mode shape MAC index measures the correlation (the similarity) between two corresponding series of modal amplitudes. Its value varies from 0 to 1: MAC = 1 means a perfect correlation, while MAC = 0 means that eigenmodes $\{\Phi_{Aj}\}$ e $\{\Phi_{Bj}\}$ are not correlated. In theory, mode shapes remain invariant if the structure do not suffer alterations, because they are structural intrinsic characteristics, depending entirely on mass and stiffness distributions. The presence of a certain damage -that is, for a fixed element, the reduction of cross-section inertial properties- involves a sensible change in those characteristics, in a way depending on damage extent and position. This is the reason why damage identification can be conducted paying attention to the differences between the two mode shapes series obtained through two different dynamic investigations performed -one after the other- on the same structure: the more MAC tends to *0*, the more the structure is damaged.

As MAC index does not take into account local deviations of displacement, it has been introduced another index, the Coordinate Modal Assurance Criterion (COMAC), expressed by the following relation [6]:

$$COMAC_{(i)} = \frac{\left(\sum_{k=j=1}^{L} \Phi_{Aj}^{i} \times \Phi_{Bk}^{i}\right)^2}{\sum_{j=1}^{L}\left(\Phi_{Aj}^{i}\right)^2 \times \sum_{k=1}^{L}\left(\Phi_{Bk}^{i}\right)^2} \tag{2}$$

where L is the total number of investigated modes and $i=1\ldots n$ is the generic point of measure. This index can be used to identify the positions in which the two series Φ_A and Φ_B of mode shapes are discordant, because it measures the correlation between all the displacements at *i*th point corresponding to the different modes. If the COMAC value is equal to 1, then no difference appears between the deflection coordinates in the intact state and the damaged state of the structure. The lower COMAC values signify the differences in coordinates and, thus, possible damage.

2.2. Mode Shapes Curvature Method

The Mode Shapes Curvature Method is based on a direct consequence of damage, that is the reduction of flexural stiffness of a structure in correspondence of the damaged parts: this result increases the amplitude of the curvature at those parts and so it can be used to detect and locate the damage (i.e. a crack). The change in curvature increases with the reduction in the value of the flexural stiffness and, therefore, the amount of damage can be obtained from the amplitude of the change in curvature [5].

To demonstrate this property it is necessary to consider the relationship between the flexural stiffness EI of a simply supported prismatic beam and the frequency ω_n of the *n*th mode of vibration:

$$\omega_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{m}} \tag{3}$$

where E is the longitudinal elastic modulus, I is the inertia of the section, L is the length of the beam and m is its mass for unit of length. The relationship between the curvature κ of the axis of the beam and the bending moment M at the cross-section considered is:

$$\frac{1}{EI} = \frac{\kappa}{M} \tag{4}$$

Substituting Eq. (1) in (2):

$$\kappa = \frac{n^4\pi^4 M}{mL^4} \times \frac{1}{\omega_n^2} \tag{5}$$

Eq. (5) shows the relationship between the curvature and the natural frequencies. This property is always valid and can be extended also to non-linear non-prismatic beams [13].

The relationship between curvature and beam deflection is then expressed by:

$$\kappa = \left[\frac{\partial^2 \upsilon}{\partial x^2}\right] \times \left[1 + \left(\frac{\partial \upsilon}{\partial x}\right)^2\right]^{-3/2} \quad (6)$$

where υ is the vertical displacement at the considered cross-section. In the field of small deflections and slope, eq. (6) becomes:

$$\kappa = \frac{\partial^2 \upsilon}{\partial x^2} \quad (7)$$

The partial derivative at second member can be computed through an approximate formula, the so-called *central difference formula*:

$$\kappa = \frac{\upsilon_{i+1} - 2\upsilon_i + \upsilon_{i-1}}{l^2} \quad (8)$$

where l is the distance between two successive measured points (or the element length of the FE model in a numerical study). In this way, the curvature of the *j*th mode shape can be obtained from the value of the vertical displacements of the same mode shape, that is from the eigenmode matrix.

Since in both methods described the whole analysis is based on modal parameters, their reliability of course depends on the accuracy of those modal parameters.

The model adopted considers the deck of the bridge as a simply supported beam divided in 21 beam elements (Fig. **1**). In this perspective an extensive testing campaign and an accurate dynamic identification will give more realistic results of the level of damage. Applications are found in [14-19] where EFDD and SSI methods and Peak Picking technique for output-only signals are used to identify the parameters of the structures.

3. NUMERICAL SIMULATION

The two methods for damage identification previously illustrated have been applied to the results of the modal analysis of a simply supported finite element bridge model. The model consists of using 21 beam elements. Damage has been simulated by reducing the stiffness *EI* of the element in the middle of the bridge [20, 21]. As a sample-structure it has been considered the I-40 bridge over the Rio Grande in Albuquerque (NM, USA). The values to assign to the mechanical characteristics of its cross-section in order to model the bridge with a series of beam elements are found in Farrar *et al.*, [22].

The model considered is shown in (Fig. **1**): the total length of the deck is 39.9 *m*, while each single element used in the mesh is 1.9 *m* length. The beam elements are connected each other through a continuous restraint. Modal analysis has been conducted by using *SAP2000* FE code [23]; the results obtained are shown in (Fig. **2**), which summarizes the mode shape of the first five bending modes of the undamaged bridge. Damage has been modeled by introducing into *element 11* a 90% reduction in *EI*; of course this high reduction in stiffness corresponds to a very strong assumption about damage extension: usually a damage around 30% is already a high damage. However, for the aim of this paper, assuming 90% of damage there is a clear change in the dynamic response of the bridge, as shown by the plots of the mode shapes in (Fig. **3**). The natural frequencies of the intact and the damaged bridge are collected in Table **1** for the first eight modes of vibration.

Even if in literature it is possible to find methods and experimental studies able to localize the existence and position of the damage [24-26] in this case the simple reading of Table **1** allows to conclude that in the bridge a certain level of damage has come up, as the natural frequencies of the second model are always lower than those of the first one.

4. CHANGE IN MODE SHAPES METHOD

MAC index has been determined from Eq.(1), in which it has been substituted to $\{\Phi_{A,j}\}$ the *j*th column of the eigenmode matrix relative to the undamaged state, to $\{\Phi_{B,k}\}$ the *k*th column of the eigenmode matrix relative to the damaged one. The results are plotted in (Fig. **4**), in which the values of the MAC index corresponding to a specific mode shape considered is reported.

From this plot it is clear that the lowest values appear at the second and fourth modes of vibration, that is $MAC_{(j=2)}$= 0.949 and $MAC_{(j=4)}$= 0.708. According to the physical meaning of MAC, this involves that the lower correlation between the mode shapes is connected with the second and fourth mode of vibration, while the other modes are substantially invariant (MAC > 0.98). This property is corroborated by the direct comparison of the plots of the mode shapes shown in (Figs. **2** and **3**). In fact, the highest difference between the mode shapes occurs at the second and the fourth modes of vibration.

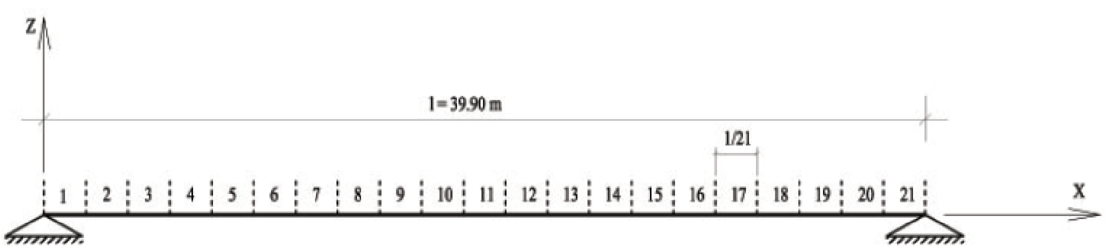


Fig. (1). FE beam model of the deck (A=5684.5 cm^2; EI=102.38×10^{12} daN×cm^2).

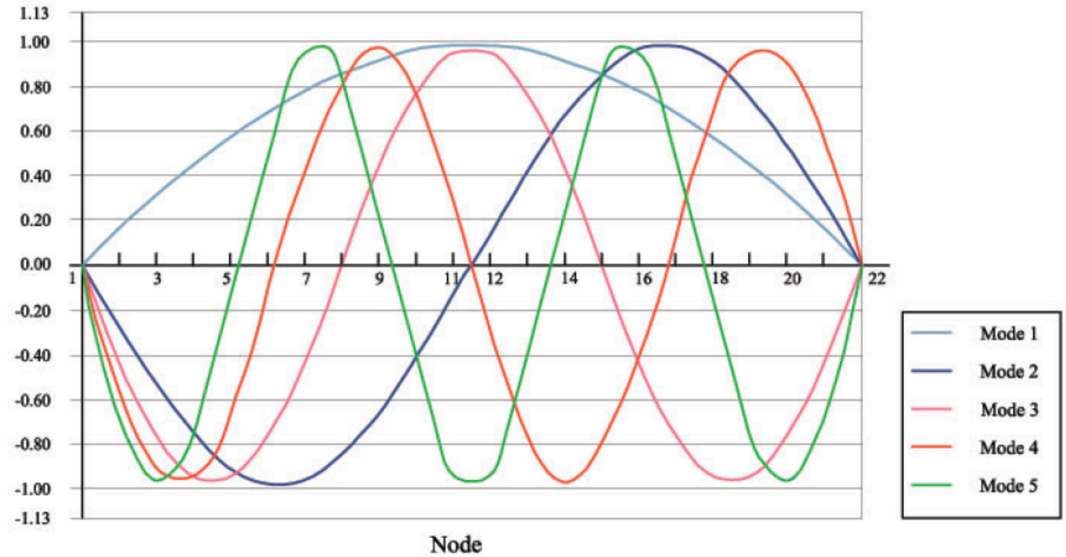


Fig. (2). First five modal shapes of the undamaged bridge.

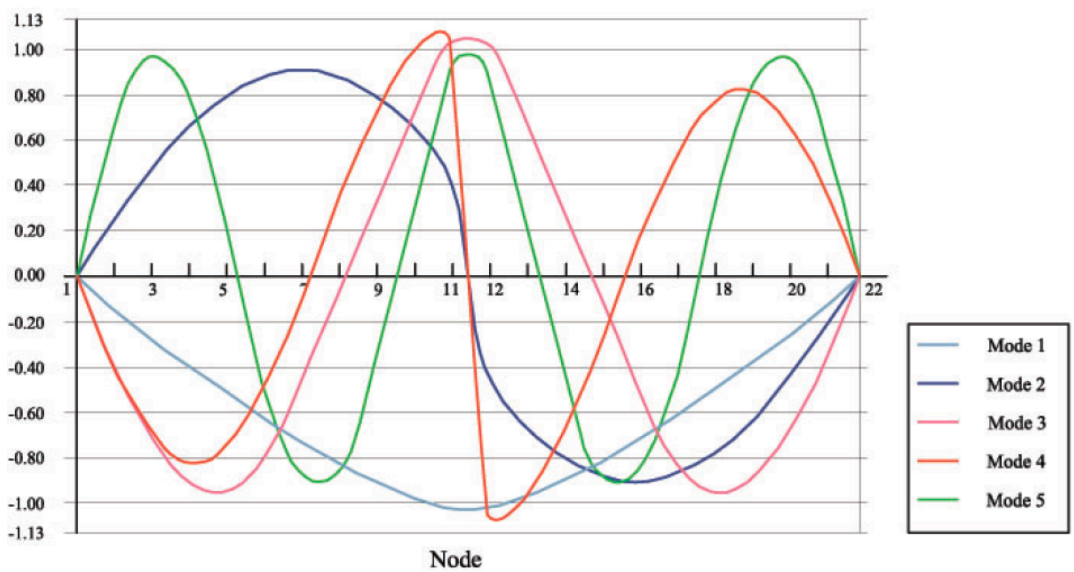


Fig. (3). First five modal shapes of the damaged bridge.

Table 1. Eigen-Frequencies of the Mode Shapes of the Deck

Frequencies (rad/s)		
Mode	**Intact bridge**	**Damaged bridges**
1°	17.84	13.48
2°	60.68	51.69
3°	112.69	102.34
4°	113.22	104.29
5°	136.53	112.93

Since that higher deviation occurs near the middle of the span, a first supposition about damage localization could be advanced. However, to obtain an analytical information (not only qualitative) about the damage identification it is necessary to determine the COMAC index. In order to do it, it is necessary to apply Eq.(2) by substituting in $\{\Phi^i_A\}$ the *i*th row of the mode shape matrix of the intact structure and in $\{\Phi^i_B\}$ the *i*th row of the analogous matrix of the damaged one. The results are plotted in Fig. (**5**): on the y-axis the values of the COMAC index corresponding to a specific cross-section considered along the axis of the deck (or the points of measurement in a real dynamic investigation) are reported.

From this plot it is clear that the lowest values appear at sections 3 and 20 and at the middle of the span: $COMAC_{(3)}$= 0.16151 and $COMAC_{(11)}$= 0.25894 (because of the symmetry the same values appear at section 20 and 12). According to the physical meaning of COMAC, this involves that the highest deviation between mode shapes occurs at sections 3, 11, 12 and 20. However, COMAC values are very low ($COMAC_{(i)} < 0.6$): this confirms that the damage, even if restricted, changes the global response of a structure.

In absence of any kind of *a priori* information regarding damage detection, the analysis of the plot of (Fig. **5**) puts in evidence that the bridge is damaged in correspondence of elements 11, 3-4 and 19-20. In reality, damage has been introduced only in *element 11*, so the information provided by COMAC is partially wrong (false-positive error).

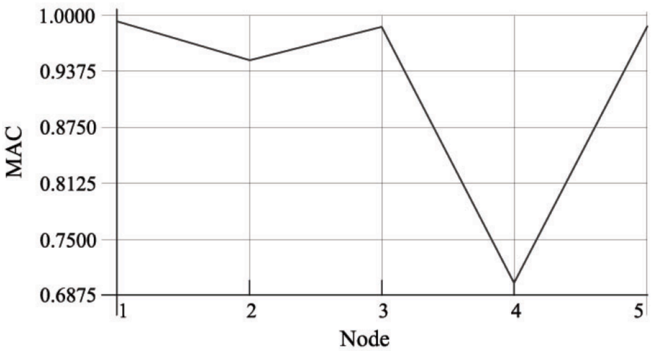


Fig. (4). MAC values for the first five bending modes.

5. MODE SHAPES CURVATURE METHOD

This method is based on the graphical representation of the absolute difference between the modal curvatures of the intact and the damaged structure: the graph obtained in this way shows a high peak at the damaged elements, indicating the presence of a fault. Again the damaged element is *element 11*, between cross-section *11* and cross-section *12*.

Modal curvatures (MC) can be calculated by using the central difference formula (Eq.(8)), where now the element size of the Finite Element (FE) adopted is $l=1.9\ m$, while υ_i is the deflection of the deck at the considered cross-section. The subsequent operation concerned with the calculation, for all modes investigated, of the following difference: $|\kappa_{int} - \kappa_{dan}|_i$, where κ_{int} is the curvature referred to the intact beam and κ_{dan} is referred to the damaged beam; i is referred to the ith cross-section. Results are plotted in (Fig. **6**) for the first four bending modes.

It can be observed that for the higher modes (especially mode 4), the absolute difference in modal curvatures shows several peaks not only at the position of the damaged element but also at different undamaged locations. This can cause confusion in all those practical applications in which the location of the damage is not known in advance.

In order to eliminate misleading information and to summarize the results for all modes, it is necessary to calculate the Curvature Damage Factor (CDF, [20]):

$$\mathrm{CDF} = \frac{1}{N} \cdot \sum_{i=1}^{N} \left| \upsilon''_{0i} - \upsilon''_{di} \right| \tag{9}$$

where N is the total number of modes considered, υ_0" is the curvature mode shape of the intact structure and υ_d" is that of the damaged structure. The results are plotted in (Fig. **7**): the damage position appears clearly at *element 11*.

The comparison between the two methods investigated in this paper shows that mode shapes curvature is a parameter more sensitive to damage than MAC and COMAC indexes. The latters, in fact, point out the damage actually introduced into the deck but also provide a false-positive error (at elements 3 and 20). This result confirms the observations in [27]: the most appropriate field of application for MAC and COMAC indexes is the validation of FE models, in which it occurs to compare the modal behavior of two different structures by measuring the correlation of the corresponding mode shapes.

The limits of the mode shapes curvature method still remain those reported in [20] in relation to the monitoring of real structures. The measured mode shapes with difficulty are regular curves, so the computed modal curvature will show irregular variation: there is the need to smooth the measured mode shapes by using curve fitting analysis before applying Eq.(8). The mode shape measurement has been utilized also to experimentally locate the position of a damage using the dynamic shape measurement, such as in [28].

6. CONCLUSIONS

Two methods proposed in literature for damage identification in bridges have been tested using simulated data for a simply supported bridge structure, the Change in Mode Shapes Method and the Mode Shapes Curvature method, based on MAC and COMAC indexes, is not properly sensitive to damage since the change in displacement mode shapes generally is too small to be used without generating false-positive errors.

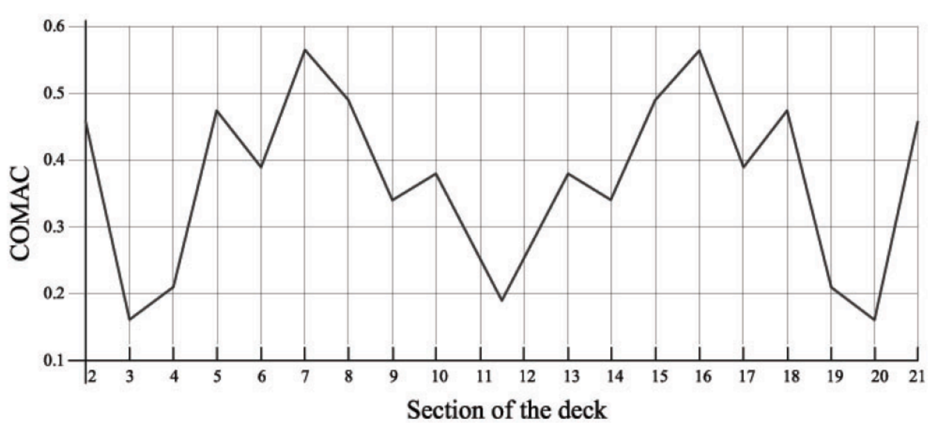
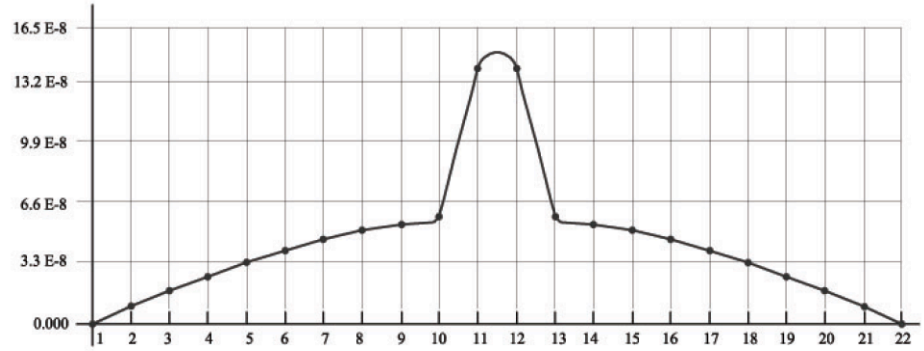
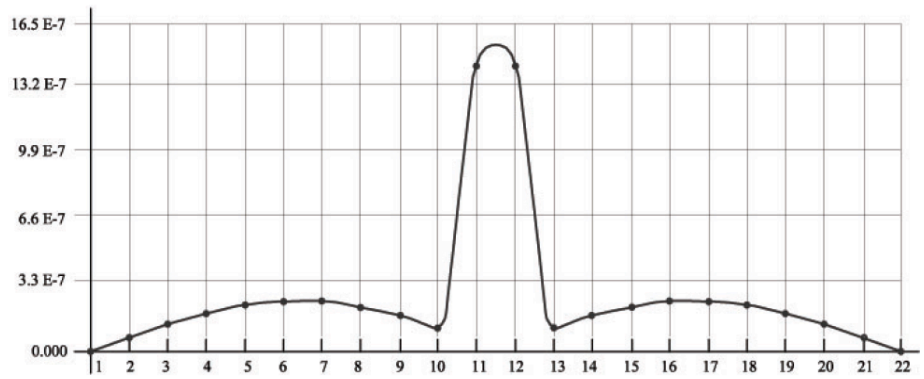


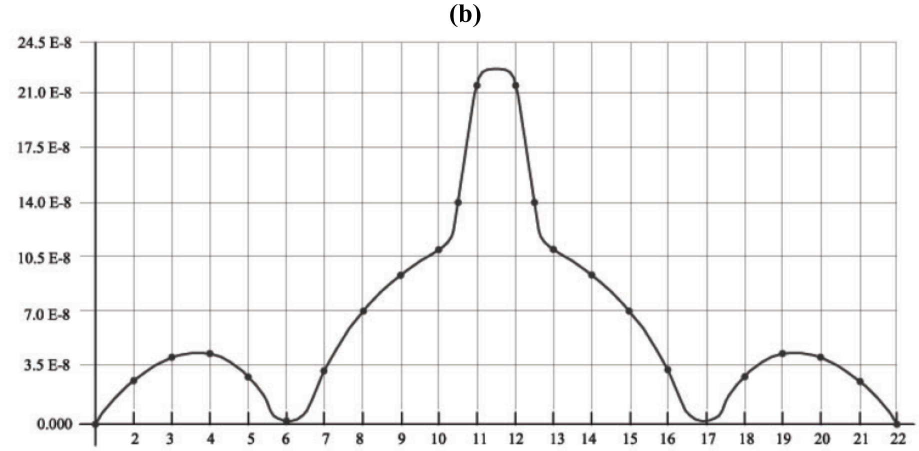
Fig. (5). COMAC values for the first five bending modes.



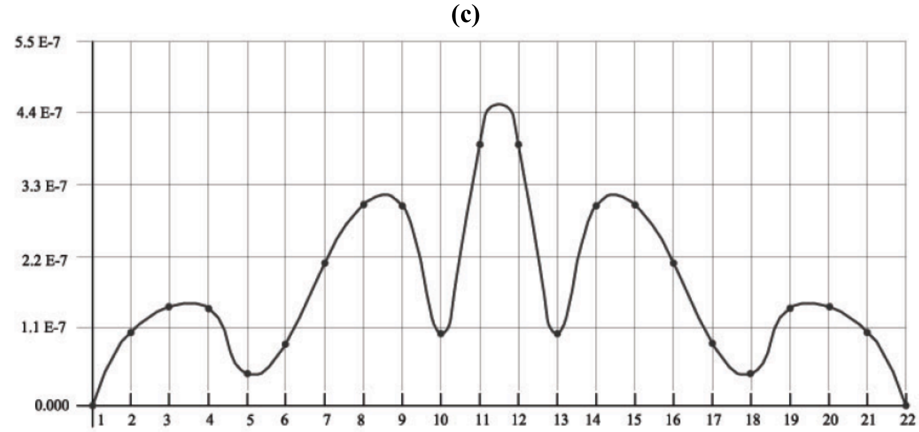
(a)



(b)



(c)



(d)

Fig. (6). Absolute value of the differences in MC corresponding to the first four bending modes. (**a**) mode 1; (**b**) mode 2; (**c**) mode 3; (**d**) mode 4.

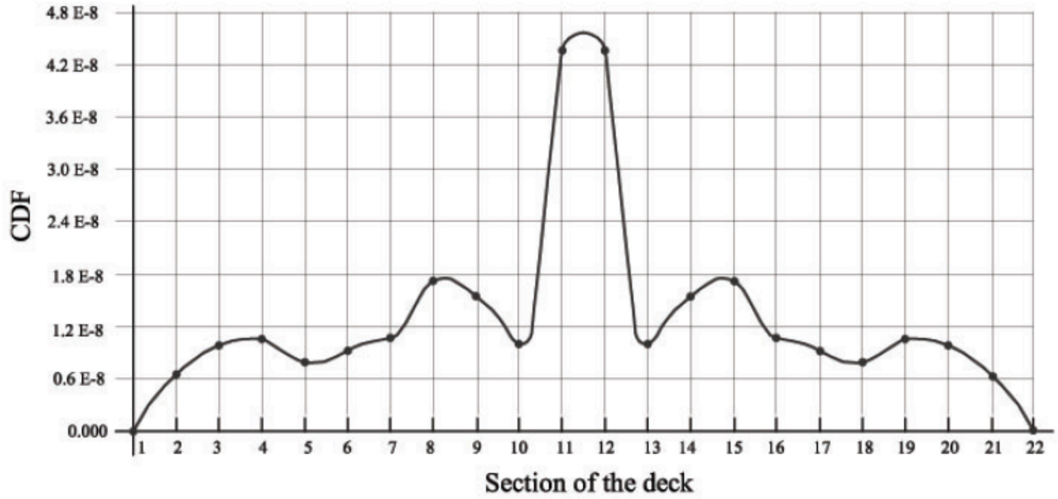


Fig. (7). Curvature Damage Factor (CDF) along beam model.

In the particular case analyzed in the present paper with a very high damage (90%) concentrated in a single section of the bridge and without any experimental noise, the Mode Shapes Curvature Method allows identifying the damage with precision, since the change in frequency is intimately related to the change in curvature. However, modal curvatures are very difficult to obtain experimentally, since the measurement accuracy is still not sufficiently elevated. Further studies should be addressed to improve the modeling of damage in a simple way (in order to calculate the dynamic properties of a damaged structures) and to develop sensors able to acquire and process modal curvatures during the monitoring of civil engineering structures.

CONFLICT OF INTEREST

The author(s) confirm that this article content has no conflicts of interest.

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