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Statics and Equilibrium Paths of Masonry Stairs

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Abstract: In the paper one refers to some research under developments by the authors on specific stairs' typologies, in particular on cantilever and helical stairs, which are largely diffused in monumental buildings. The paper responds to professional practice which demands clear and sharp models for structural analysis and calculations, and simplified approaches for correctly interpreting their structural behaviour, and also for providing some indications supporting the design of proper refurbishments.

Keywords: Masonry, Stairs, Vaults, Low tensile resistance, No Tension models, Helical stair, Cantilever stair, Equilibrium.

PROBLEMS IN TREATING MASONRY STAIRS AND POSSIBLE APPROACHES

The significant importance of studying ancient buildings is especially felt by researchers from European countries, where the monumental and historical heritage is largely diffused on the territory [1-5]. As well known, studying masonry structures is much different from studying reinforced concrete constructions where analyses may be developed by means of commercial software.

Masonry structures are rather complex systems where in most cases it is not sufficient to study the behaviour of the single structural element as in concrete structures. The masonry material exhibits a non conventional behaviour that must be investigated for the specific case in hand in order to formulate the most generic model.

One of the sources of complexity is often represented by the great variety of masonry typologies, usually made of materials with different mechanical characteristics, coupled to a variegated arrangement constituting the texture of the masonry. The texture significantly influences the overall behaviour of the structure representing the major problem in the formulation of an adequate and reliable structural model, at the meanwhile univocal and complete. On the other hand the variety of textures existing in masonry buildings pushes towards the elaboration of flexible theoretical models which can be adapted to the variety of possible situations.

In masonry buildings it is possible to recognize and isolate the basic structural elements (as e.g. vaults, arches, walls), similarly to the homolog elements of a modern structure (as e.g. floors, beams, pillars). Nevertheless the operation of the basic masonry elements is rather more complex than in other civil buildings. Moreover the constituent elements' interaction is often not so evident in their real hierarchical degree of connection and relationship. That one represents a problem for setting up an efficient structural model of masonry buildings.

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These and other specific characteristics make the masonry so complex [6-29] to be studied and make the results deriving from a large part of the software not reliable in the practice.

For many tens of years the research group of the authors have been developing an extended research about masonry fabrics and the relevant problematic situations, by formulating original theoretical and experimental approaches (see e.g. [16-28]), also with reference to the optimal interventions of refurbishment also made by FRP provisions (see e.g. [30, 31]). It represents one of the most large and complete existing researches about the masonry behaviour, started during early 80s, which has deeply contributed to this complex subject.

In most recent times, a special attention has been devoted by the research group to the study of problems concerning masonry stairways, which represents a problem rarely treated in literature [32-34], since most of researches focus on the stairs made of material supporting tensile stresses [35-44].

A specific study about the analysis of the stress field distribution and the equilibrium path has been developed on two particular types of masonry stairways (Figs. **1** and **2**) such as the "*half-barrel*" or "*cantilever*" or "*Roman*" stairways and the "*helicoidal*" or "*spiral*" stairways. These typologies are largely diffused in Italian ancient and monumental masonry buildings (especially since the XVI century buildings) and for their complexity, these structures, or similar ones, are usually studied in a three-dimensional space with the support of complex software. Nevertheless the professional practice demands clear and sharp models for structural analysis and calculation, and simplified approaches often help the interpretation of the structural behaviour and suggest procedures for decision and design in the emergency or in the common refurbishment of a masonry fabric.

The static interpretation of the Roman stairs [45, 46], for example, requires some complex analyses, which in many cases cannot be summarized in a controllable behaviour for certifying its static suitability. However on the basis of some considerations it is possible to set up a simplified method for an approximate check of a cantilever stair in order to facilitate its analysis. Thereforea simplified approach for analysing

Fig. (1). Examples of monumental spiral staircases.

Fig. (2). Examples of monumental cantilever staircases.

the Roman stairways' behaviour has been developed by checking the equilibrium mechanism of the vaulted stairways through a classic "beam model", starting from the assumption that the masonry is not able to resist tensile stresses and that a no-tension (or NT) model is assumed for the material. Then the stress fields which are in equilibrium with the self-weight and live loads, and compatible with the resistant skills of the masonry material, have been searched for in order to get the stress solution.

In the static analysis a simplified model can consider the Roman staircase as the composition of some basic components: the landing, the angle connection, and the flight of stairs (two or three depending on the structure morphology). The structure is globally supported by the outside walls system which represents the stair box.

The landing can be easily modelled by means of the arch model. Analogously the angle connection can be analyzed as a quarter of a trough-vault, which is stable provided that the converging flights and/or landing are stable. So the resulting forces are transmitted to the flight of the stairs, which becomes the central problem in the equilibrium. The drift of the flight with respect to the landing yields a longitudinal compression which is associated to an arch effect in the flight thickness helping its equilibrium. Nevertheless the profile of the flight is usually rectilinear, which does not help the arch effect in the longitudinal direction, and the vertical reaction of the flight is concentrated at the connection point of the flight with the outside wall.

After all, the flight of stairs can be viewed at like a horizontally wide and tilted beam, which is equipped with a lateral continuous support at one edge of its cross section, subject to a vertical load. The vertical load, given by the superposition of the permanent and the accidental loads, presents an eccentricity that activates a reaction in the flight of the stairs. Therefore the torsional load is distributed on the flight's length and the reactive torsional moments are activated in order to equilibrate it in correspondence of the angle connections.

If the flight of stairs is able to completely absorb the activated torsion, the equilibrium can exist also without resorting to other mechanisms of maintenance, like e.g. the arch effect in the lengthwise direction. Another way to balance the "cantilever" is given by an horizontal shear reaction. In order to adapt itself to one more favourable distribution of tensions the flight activates a arch-type effect represented by a stress curve included into the profile of the flight, but with a non-conventional shape hardly analyzable with elementary methods. The torsion stress activates also additional reactions in correspondence of the angle connections which take a part to advantage of the jam improving the resistance skill of the mortar joint – rows.

Moreover some instructions for planning the optimal application and distribution of the reinforcement can be inferred on the basis of the developed analysis and of the knowledge of the stairs behaviour.

As regards the helical stair typology shown in (Fig. **1**), because of its appearance and save-spacing skill, this staircase typology is nowadays increasingly getting popular worldwide and a renewed interest is attracted in the area of analysis and design of helicoidal stair slabs, mainly focusing on problems related to its complex geometrical configuration.

If the spiral form is considered as a three dimensional (3-D) helical girder and is reduced to its middle line having the same stiffness as that of the original structure, the slab action of helicoids is neglected and it is assumed that the bending stiffness and torsional stiffness of a warped girder are the same as those of a straight beam, thus achieving an helical girder solution for helicoidal stairs, which takes into account the essential 3-D characteristics of helicoid and its inherent structural efficiency.

Attempts of analysing the internal forces due to dead and live loads in fixed ended circular stairs having an intermediate landing are recorded [39] where the influence lines of the structure modelled as a linearly elastic member in space are drawn at various cross sections. More recent studies report about parametric analyses and design charts in case of intermediate landings [40], including a conspicuous literature about FEM analysis developed under the elastic assumption (see e.g. [41-44]).

Helicoidal staircases are also very often encountered in ancient and/or monumental masonry buildings. Actually the interest of this typology lays in the wide variety of spiral stairs constructed since the classical antiquity in the Greco-Roman period, dating back to the Trajan's Column built in the 5th century B.C: (an interior stairway develops in the column itself above the pedestal) and handed down until nowadays, permanently hold during the different historical periods [47, 48]. Besides the first attempts of studying their static operation (see e.g. [49] where the reason for structural malfunctions is identified in the poor tension capacity of masonry) most of the studies rely upon a number of rather significant simplifications, and are mostly referred to the case of stairs made by an elastic homogeneous and isotropic material.

One should emphasize that a correct structural approach to the study of the masonry buildings and constructions should be founded on the acquisition of all the scientific and technological elements and data, involving those originally used to design, or more properly conceive, the structure, the knowledge of the territory's organization where the structure is placed, and possibly rebuilding the historical, anthropological and environmental changes undergone by the fabric throughout its existence. The masonry structure is a rather complex system not only owed to the use of different materials but especially because of the organization of these materials in the structure, that is to say the texture. The texture influences in a significant way the overall behaviour of the structure representing a relevant problem in the formulation of the structural behaviour model, both univocal and complete. On the other hand the variety of textures existing in masonry buildings pushes towards the elaboration of sufficiently flexible models which can be effectively adapted to the usually known schemes of masonry elements. Usually some assumptions are adopted for a simplified approach to the problem. The first of them consists of the mechanical hypothesis that the masonry is a Low-Tension (LT), which means that the structure tends to equilibrate the loads annealing high tension stresses, possibly by developing a set of suitable fractures. In the limit, masonry can be assimilated to a No-Tension (NT) resistant material, which means a material with zero tensile resistance and a linear elastic behaviour in compression [16-23], thus drastically simplifying the need for data about tensile strength, that is in most cases inessential for statics.

Under this assumption the existence of equilibrium states, or alternatively of collapse mechanisms, is then investigated setting up the relevant problems and searching for the related solutions, which can also be successfully used for non-cohesive materials [27, 28]. Usually, due to the non-linear mechanical behaviour of the masonry structures, which is often coupled to the complex geometry of its structural components, these components are analysed one by one.

The authors have produced in the last decades many scientific papers [16-23] on the subject, which represent a research stream of primary interest because of the many existing historical and monumental masonry constructions and still need more contributions for the deepening of many features and the development of still lacking adequate tools of analysis under the static and dynamic regimes, where vulnerability assessment is required also for planning adequate preservation and modern control strategies [50-52] particularly important in absence of reliable forecast about the seismic action [53, 54], even with reference to pure rocking modes [55, 56] of single members of the structure, such as walls or staircases.

One should consider that few efforts are available in literature aimed at addressing the analysis of spiral staircases under the assumption that the material is unable to resist significant tension and bending [49, 57] and that elastic analyses, when forced to comply with structures working under these conditions in practice, ends in misleading results and erroneous conclusions about the interpretation of their static operation.

On the other side, one must consider that one characteristic feature of masonry spiral staircases is that it is embedded in a boundary chamber and so the flight is laterally supported by the wall on the whole length. Moreover most often the masonry texture is organized according to a smart pattern (see e.g [58]), which provides the ideal belt with some capacity to resist torque stress thanks to the seizure of the bricks (see e.g. [59]).

Approaches are developed by the authors for providing closed-form solutions for the equilibrium of stairs, including both active and reactive, force and moment components, opening the way to a complete overview of the multi-variate equilibrium attitudes the structure can potentially assume. In this sense, the result is valuable also for masonry helicoidal staircases, allowing to search for LT or even NT solutions, and thereafter to investigate the reasons why even ancient staircases stand up, but also outlining an approach to the problem that can be extended to different shape typologies.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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REFERENCES

[1] C.A. Syrmakezis, and P.G. Asteris, "Masonry failure criterion under biaxial stress state", *Journal of Materials in Civil Engineering*; *American Society of Civil Engineers (ASCE)*, vol. 13, no. 1, pp. 58-64, 2001.

[2] P.B. Lourenço, "Computations on historic masonry structures", *Progress in Structural Engineering and Materials*, vol. 4, no. 3, pp. 301-19, 2002.

[3] P.G. Asteris, A.D. Tzamtzis, P.P. Vouthouni, and D.S. Sophianopoulos, "Earthquake resistant design and rehabilitation of masonry historical structures", *Practice Periodical on Structural Design and Construction*, *American Society of Civil Engineers (ASCE)*, vol. 10, no. 1, pp. 49-55, 2005.

[4] P.G. Asteris, "On the structural analysis and seismic protection of historical masonry structures", *The Open Construction and Building Technology Journal*, vol. 2, pp.124-133, 2008.

[5] P.G. Asteris, and A.D. Tzamtzis, "Nonlinear seismic response analysis of realistic gravity dam-reservoir systems", *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 4, no. 4, pp. 329-338, 2003.

[6] J. Heyman, "The stone skeleton", *Journal of Solids and Structures*, vol. 2, pp. 269-279, 1966.

[7] J. Heyman, "The safety of masonry arches", *Journal of Mechanic Sciences*, vol. 2, pp. 363-384, 1969.

[8] A. Baratta, "Statics and reliability of masonry structures", In: *General Principles Applications in Mechanics of Solids and Structures*. CISM, Udine, pp. 205-235, 1991.

[9] A.Baratta, M.Vigo, and G.Voiello, "Calcolo di archi in materiale non resistente a trazione mediante il principio del minimo lavoro complementare". *Proceding of 1st Natural. Conference. ASS.I.R.C.CO*. Verona, 1981.

[10] Z.P. Bazant, "Analysis of work-of fracture method for measuring fracture energy of concrete", *Journal of Engineering Mechanics ASCE*, vol. 122, no. 2, pp. 138-144, 1996.

[11] A. Baratta, "Il materiale Non Reagente a Trazione come modello per il calcolo delle tensioni nelle pareti murarie", *Journal Restauro*, vol. 75, pp. 53-77, 1984.

[12] M. Como, and A. Grimaldi, "A unilateral model for limit analysis of masonry walls", In: *Unilateral Problems in Structural Analysis*, Ravello, pp. 25-46, 1983.

[13] Z.P. Bazant, and Y.N. Li, "Stability of cohesive crack model: Part I: Energy principles", *Journal of Applied Mechanics*. vol. 62, no. 12, pp. 959-964, 1995.

[14] R.W. Odgen, *Non linear elastic deformations.* Mineola, New York, Dover Publications Inc., 1997.

[15] A.M. Khludnev, and V.A. Kovtunenko, "Analysis of cracks in solids", *Applied Mechanics Review*, vol. 53, no. 10, 2000.

[16] A. Baratta, "Strength capacity of No Tension portal arch-frame under combined seismic and ash loads", *Journal of Volcanological and Geothermal Research*, vol. 133, no. 1-4, pp. 369-376, 2004. ISSN: 0377-0273, DOI: 10.1016/S0377-0273(03)00408-6.

[17] A. Baratta, and O. Corbi, "On variational approaches in NRT continua", *International Journal of Solids and Structures, Elsevier Science*, vol. 42, pp. 5307-5321, 2005. ISSN: 0020-7683. DOI: 10.1016/j.ijsolstr.2005.03.075.

[18] A. Baratta, and O. Corbi, "An approach to masonry structural analysis by the no- tension assumption-Part I: material modeling, theoretical setup, and closed form solutions". *Applied Mechanics Reviews*, vol. 63, no. 4, pp. 040802-1/17, 2010. ISSN 0003-6900, DOI:10.1115/1.4002790.

[19] A. Baratta, and O. Corbi, "An approach to masonry structural analysis by the no-tension assumption-Part II: load singularities, numerical implementation and applications". *Applied Mechanics Reviews*, vol. 63, no. 4, pp. 040803-1/21, 2010. ISSN: 0003-6900, DOI: 10.1115/1.4002791.

[20] A. Baratta, and O. Corbi, "Relationships of L.A. theorems for NRT structures by means of duality". *International Journal of Theoretical and Applied Fracture Mechanics*, *Elsevier Science*, vol. 44, pp. 261-274, 2005. ISSN: 0167-8442. DOI: 10.1016/j.tafmec.2005.09.008.

[21] A. Baratta, and O. Corbi, "Duality in non-linear programming for limit analysis of NRT bodies", *Structural Engineering and Mechanics*, vol. 26, no. 1, pp. 15-30, 2007. ISSN: 1225-4568.

[22] A. Baratta, and O. Corbi, "On the equilibrium and admissibility coupling in NT vaults of general shape", *International Journal of Solids and Structures*, vol. 47, no. 17, pp. 2276-2284, 2010. ISSN: 0020-7683. DOI: 10.1016/j.ijsolstr.2010.02.024.

[23] A. Baratta, I. Corbi, and O. Corbi, "Stress analysis of masonry structures: Arches, walls and vaults", *Proceedings of the 6th International Conference on Structural Analysis of Historic Construction: Preserving Safety and Significance, SAHC08*; Bath; 2 July 2008-4 July 2008; Code 83644, vol. 1, pp. 321-329, 2008. ISBN: 0415468728; 978-041546872-5.

[24] A. Baratta, and O. Corbi, "On the statics of No-Tension masonry-like vaults and shells: solution domains, operative treatment and numerical validation", *Annals of Solid and Structural Mechanics*, vol. 2, no. 2-4, pp. 107-122, 2011. ISSN: 0965-9978. DOI: 10.1007/s12356-011-0022-8.

[25] A. Baratta, I. Corbi, and S. Coppari, "A method for the evaluation of the seismic vulnerability of fortified structures", *Final Conference on COST Action C26: Urban Habitat Constructions under Catastrophic Events; Naples*, Code 89987, pp. 547-552, 16 -18 September 2010. ISBN: 978-041560685-1.

[26] A. Baratta, I. Corbi, O. Corbi, and D. Rinaldis, "Experimental survey on seismic response of masonry models", *Proceedings of the 6th International Conference on Structural Analysis of Historic Constructions: Preserving Safety and Significance, SAHC08*, 2008,

Bath; Code 83644, vol. 2, pp. 799-807, 2-4 July 2008. ISBN: 0415468728; 978-041546872-5.

[27] A. Baratta, and I. Corbi, "Plane of elastic non-resisting tension material under foundation structures", *International Journal for Numerical and Analytical Methods in Geomechanics,* J. Wiley & Sons Ltd., vol. 28, pp. 531-542, 2004. ISSN: 0363-9061, DOI: 10.1002/nag.349.

[28] A. Baratta, and I. Corbi, "Spatial foundation structures over no-tension soil", *International Journal for Numerical and Analytical Methods in Geomechanics*, Wiley Ed. vol. 29, pp. 1363-1386, 2005. ISSN: 03639061, DOI: 10.1002/nag.464.

[29] D. Foti, M. Diaferio, N.I. Giannoccaro, M. Mongelli, and P.Andersen, "Operational Modal Analysis of a Historical Tower in Bari", *Conference Proceedings of the Society for Experimental Mechanics Series*, "IMAC XXIX", vol. 7, pp. 335-342, Jacksonville, Florida, USA, Jan.-3 Feb. 2011. DOI: 10.1007/978-1-4419-9316-8_31, 31.

[30] A. Baratta, and I.Corbi, "Optimal reinforcement for No-Tension structure" In: F. Ubertini, E. Viola, S. de Miranda, G. Castellazzi (eds) *Atti del 20° Convegno Nazionale di Meccanica Teorica e Applicata AIMETA*, Publi&Stampa Edizioni, Conselice (Ra), paper n. 249, pp. 10, 12-15 Settembre 2011. ISBN: 978-88-906340-1-7.

[31] F. Ascione, "Ultimate behaviour of adhesively bonded FRP lap joints", *Journal of Composites Part B, Engineering*, vol. 40, pp. 107-115, 2009. ISSN: 1359-8368, doi: 10.1016/j.compositesb.2008.11.006.

[32] S. Price, and H. Rogers, "Cantilever stone stairs", *Structural Engineer*, vol. 87, no. 11, pp. 20, 2009.

[33] P. Little, M. Hough, and E. Mullarkey, "Stone cantilever stairs - Inspection and analysis of cantilever stairs", *Structural Engineer*, vol. 87, no. 8, pp. 26-33, 2009.

[34] T. Lucas, "Cantilever stairs", *Structural Engineer*, vol. 86, no. 8, p. 28, 2008.

[35] M.Y.H. Bangash, and T. Bangash, *Staircases - Structural Analysis and Design*, Taylor & Francis: UK, p. 348, 1999.

[36] C.E. Reynolds, and J.C. Steedman, *Reinforced Concrete Designers Handbook*. 10th Edition, E and F N Spon: London, 1988.

[37] V.A. Morgan, "Comparison of analysis of helical staircases", *Concrete*, vol. 55, no. 3, pp. 127-132, 1960.

[38] A. C. Scordelis. "Closure to discussion of internal forces in uniformly loaded helicoidal girder", *ACI Journal Proceedings*, vol. 56, no. 6, Part 2, pp. 1491-1502, 1960.

[39] A.S. Arya, and A. Prakash, Analysis of Helicoidal Staircases with Intermediate Landing in Analysis of Structural System for Torsion, SP 35, American Concrete Institute, Detroit, MI, 1973.

[40] Z. Wadud, and S. Ahmad, "Simple design charts for helicoidal stair slabs with intermediate landings", *IE Journal CV,* vol. 85, pp. 269-275, 2005.

[41] J. Baorong and L. Yongfeng, *Finite Element Calculation of a Plate Steel Spiral Stair*, Structural Engineers: China, 2008.

[42] H. Xue-song, and F. Li-mei, "Application of ANSYS in structure design of reinforced concrete spiral stairs", *Journal of Tianjin Institute of Urban Construction*, 2006.

[43] W. Busool, and M. Eisenberger, "Exact static analysis of helicoidal structures of arbitrary shape and variable cross section", *Journal of Structural Engineering*, vol. 127, no. 11, pp. 1266-1275, 2001.

[44] A.F.M.S. Amin, and S. Ahmad, "Thick shell finite elements in the analysis of helicoidal stair slabs", *Proceedings of Civil and Environmental Engineering Conference New Frontiers and Challenges*, Bangkok, Thailand, 8-12 November 1999.

[45] A. Baratta, and I. Corbi, "On masonry vaulted stairs: statics and FRP reinforcement". Atti del 3° Convegno Nazionale Meccanica delle strutture in muratura rinforzate con compositi modellazione, sperimentazione, progetto, controllo (MuRiCo3), A. Di Tommaso (Ed.), pp. 51-58, Venezia, 22-24 April 2009. ISBN: 88-371-1771-X.

[46] A. Baratta, and I. Corbi, "Masonry vaulted staircases: interpretation of equilibrium paths", *Proceedings of the Eleventh International Conference on Computational Structures Technology* CST2012, Dubrownik, Civil-Comp Press, Stirlingshire, UK, paper 551, pp. 1-11, 4-7 September 2012. ISSN: 1759-3433, DOI: 10.4203/ccp.99.127.

[47] M. Beckmann, "The 'columnae coc(h)lides' of trajan and marcus aurelius", *Phoenix,* vol. 56, no. 3/4, pp. 348-357, 2002.

[48] M.W. Jones, "One hundred feet and a spiral stair: the problem of designing trajan's column", *Journal of Roman Archaeology* , vol. 6, pp. 23-38, 1993.

[49] J. Heyman, "The mechanics of masonry stairs", *Transactions of the Built Environment*, vol. 15, WIT Press, 1995.

[50] A. Baratta, and O. Corbi, "Dynamic Response and Control of Hysteretic Structures", *International Journal of Simulation Modeling Practice and Theory (SIMPAT), Elsevier Science*, vol. 11, pp. 371-385, 2003. ISSN: 1569-190X, DOI: 10.1016/S1569-190X(03)00058-3.

[51] A. Baratta, and I. Corbi, "Optimal design of base-isolators in multi-storey buildings", *International Journal of Computers & Structures, Elsevier*, vol. 82, no. 23-26, pp. 2199-2209, 2004. ISSN: 00457949 , DOI: 10.1016/j.compstruc.2004.03.061.

[52] A. Baratta, and O. Corbi, "On the dynamic behaviour of elastic-plastic structures equipped with pseudoelastic SMA reinforcements", *Journal of Computational Materials Science*, vol. 25, no. 1-2, pp. 1-13, 2002. ISSN: 09270256, DOI: 10.1016/S0927-0256(02)00245-8.

[53] A. Baratta, and I. Corbi "Epicentral distribution of seismic sources over the territory", *International Journal of Advances in Engineering Software, Elsevier*, vol. 35, no. 10-11, pp. 663-667, 2004. ISSN: 0965-9978, DOI: 10.1016/j.advengsoft.2004.03.015.

[54] A. Baratta, and I. Corbi "Evaluation of the hazard density function at the site", *International Journal of Computers & Structures, Elsevier*, vol. 83, no. 28-30, pp. 2503-2512, 2005. ISSN: 0045-7949, DOI: 10.1016/j.compstruc.2005.03.038.

[55] A. Baratta, and O. Corbi, "Analysis of the dynamics of rigid blocks using the theory of distributions", *Advances in engineering Software*, vol. 44, no. 1, pp. 15-25, 2012. ISSN: 09659978CODEN, DOI: 10.1016/j.advengsoft.2011.07.008.

[56] A. Baratta, I. Corbi, and O. Corbi, "Rocking motion of rigid blocks and their coupling with tuned sloshing dampers", in B.H.V. Topping, L.F. Costa Neves, and R.C. Barros, (Editors). In: *Proceedings of the 12th International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, UK, Paper 175, 2009. ISBN: 978-1-9050-88-32-4, DOI: 10.4203/ccp.91.175.

[57] A. Baratta, and P. Belli, "Sulla statica delle scale elicoidali in muratura", *IV Congresso Nazionale Consolidamento e recupero dell'architettura tradizionale: dagli interventi singoli agli interventi d'insieme urbano, ASSI.R.C.CO*. 1992.

[58] A. Barbieri A. Di Tommaso, and R. Massarotto, "Helical masonry vaulted staircase in Palladio and Vignola's architectures", *Proceedings of the 1st International Congress on Construction History*, Madrid, ed. S. Huerta, Madrid: I. Juan de Herrera, SEdHC, ETSAM, A. E. Benvenuto, COAM, F. Dragados, pp. 299-312, 20-24 January 2003.

[59] A. Baratta, and I. Corbi, "On the Statics of Masonry Helical Staircases", in B.H.V.Topping, Y. Tsompanakis, (Editors), *Proceedings of the 13th International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, UK, Crete; Paper 59, 6-9 September 2011. ISBN: 978-190508845-4, DOI:10.4203/ccp.96.59.

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